Critical effects of overlapping of connectivity and dependence links on percolation of networks

李明
中国科学技术大学　近代物理系
导师：汪秉宏 教授
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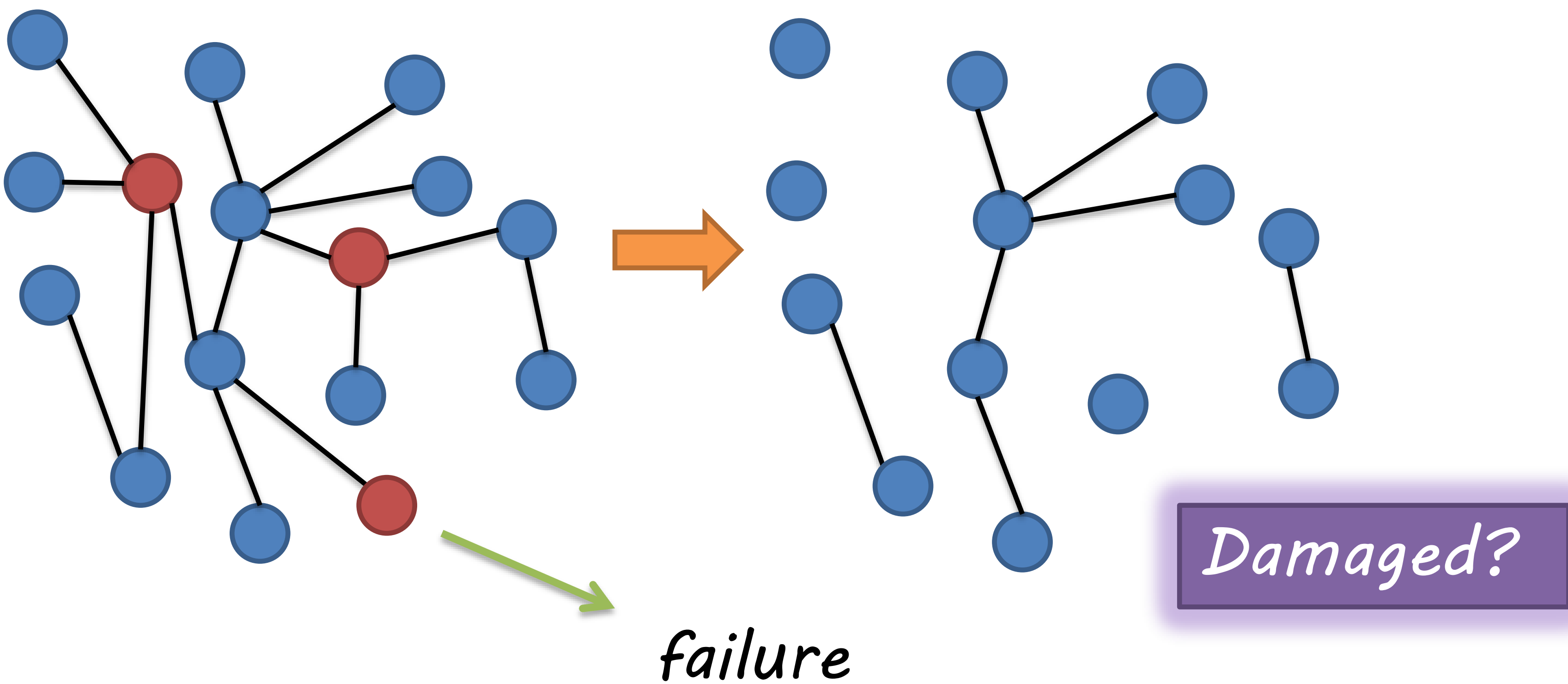
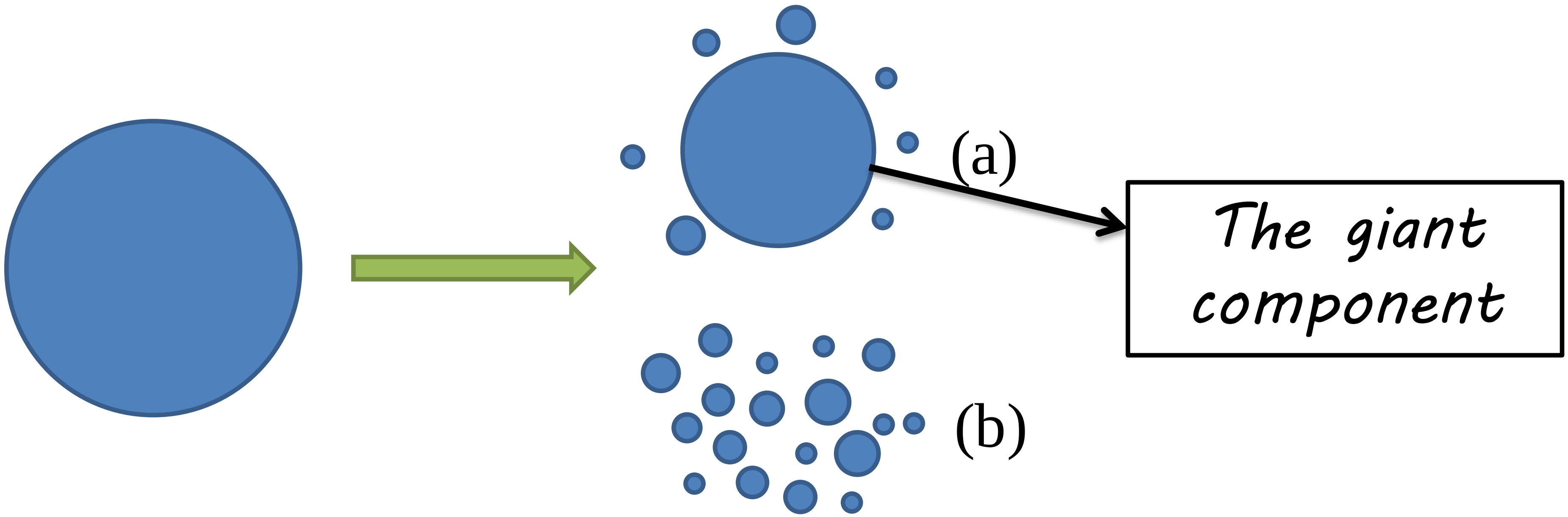
1. Background

The robustness of the networks against random failure and intentional attack.



2. Percolation on networks

For network science, percolation theory explains the existence of the giant component of a network after a fraction $1 - p$ of nodes are removed.

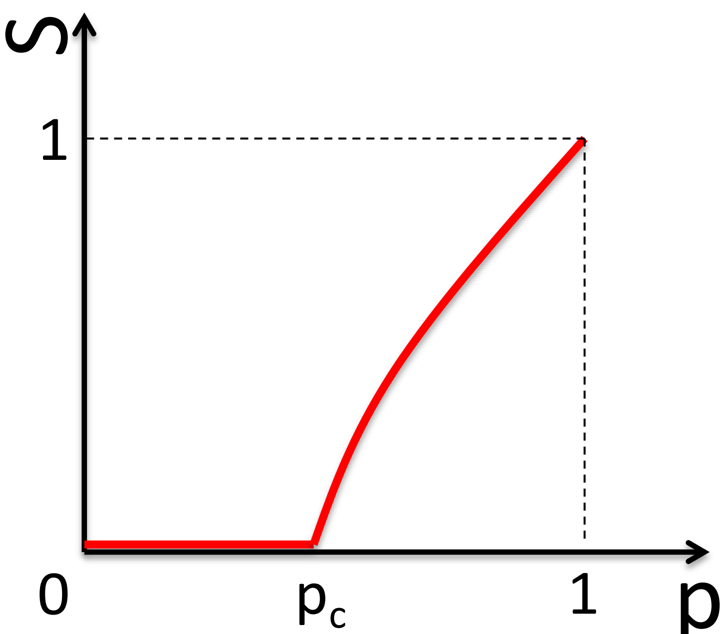


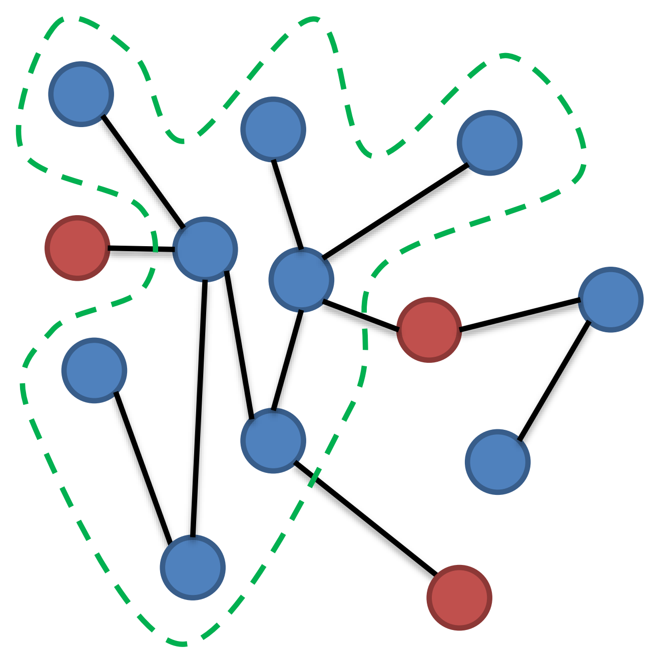
Considered as a percolation model

order parameter: *the size of the giant component i.e., a randomly chosen node belongs to the giant component*

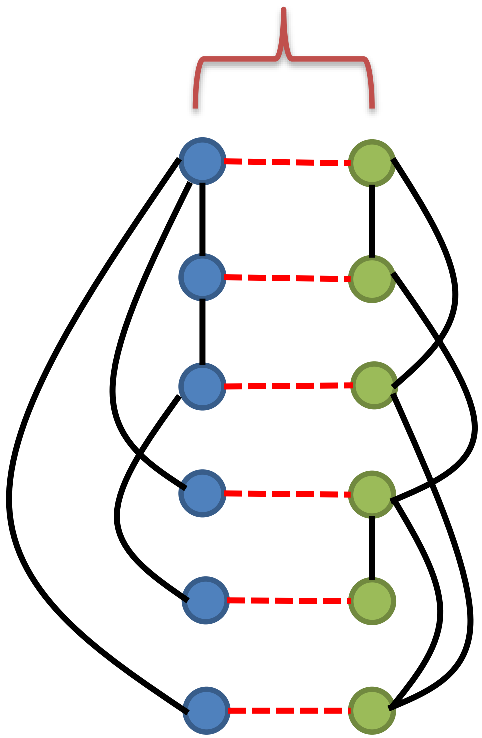
Second order phase transition

(Phys. Rev. Lett. 85, 5468 (2000)， Phys. Rev. Lett. 86, 3682 (2001))

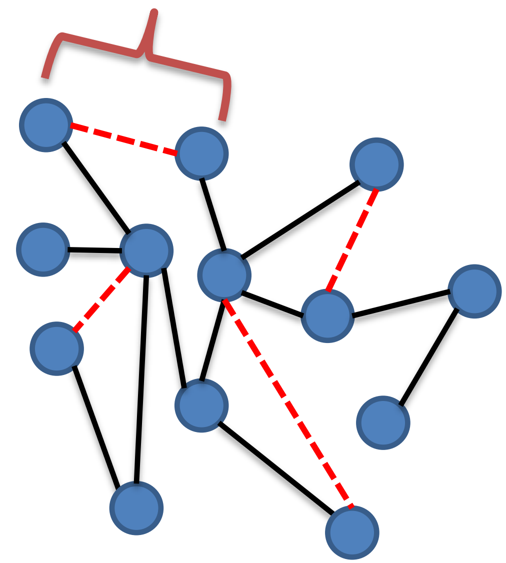




3. Percolation on networks with dependence links

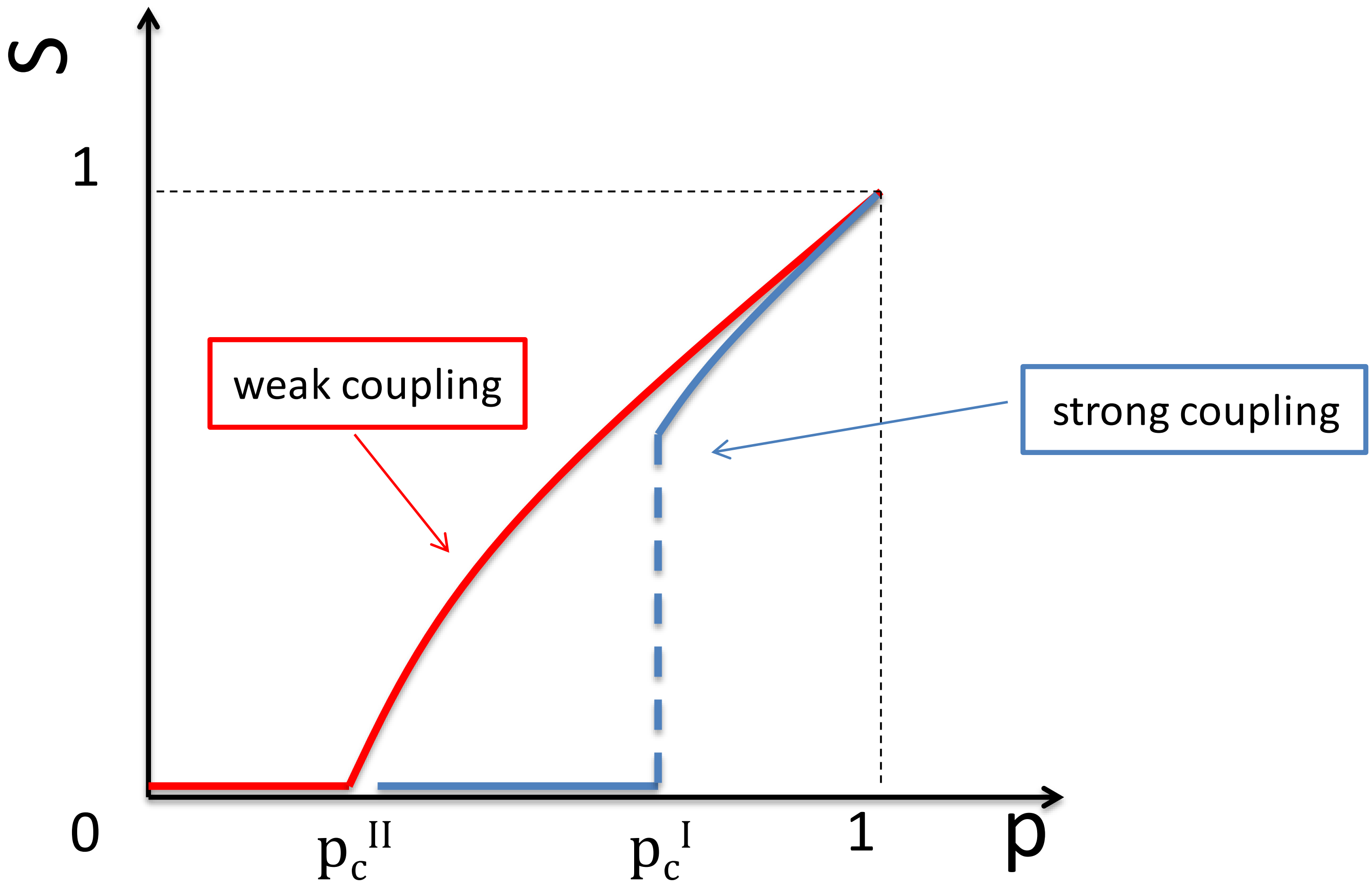


Nature 464, 1025 (2010)



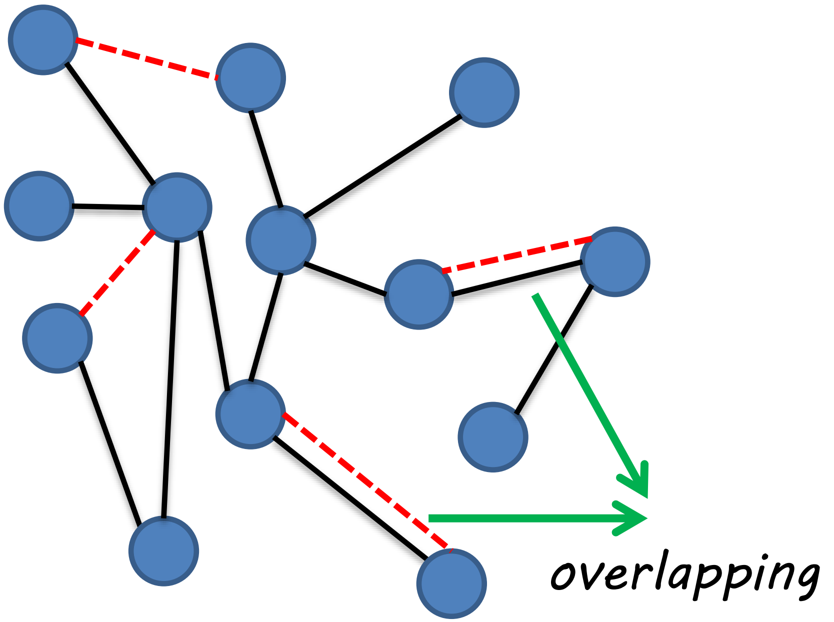
Proc. Natl. Acad. Sci. 108, 1007 (2010)

First order phase transition



4. Our work: Effects of overlapping of connectivity and dependence links

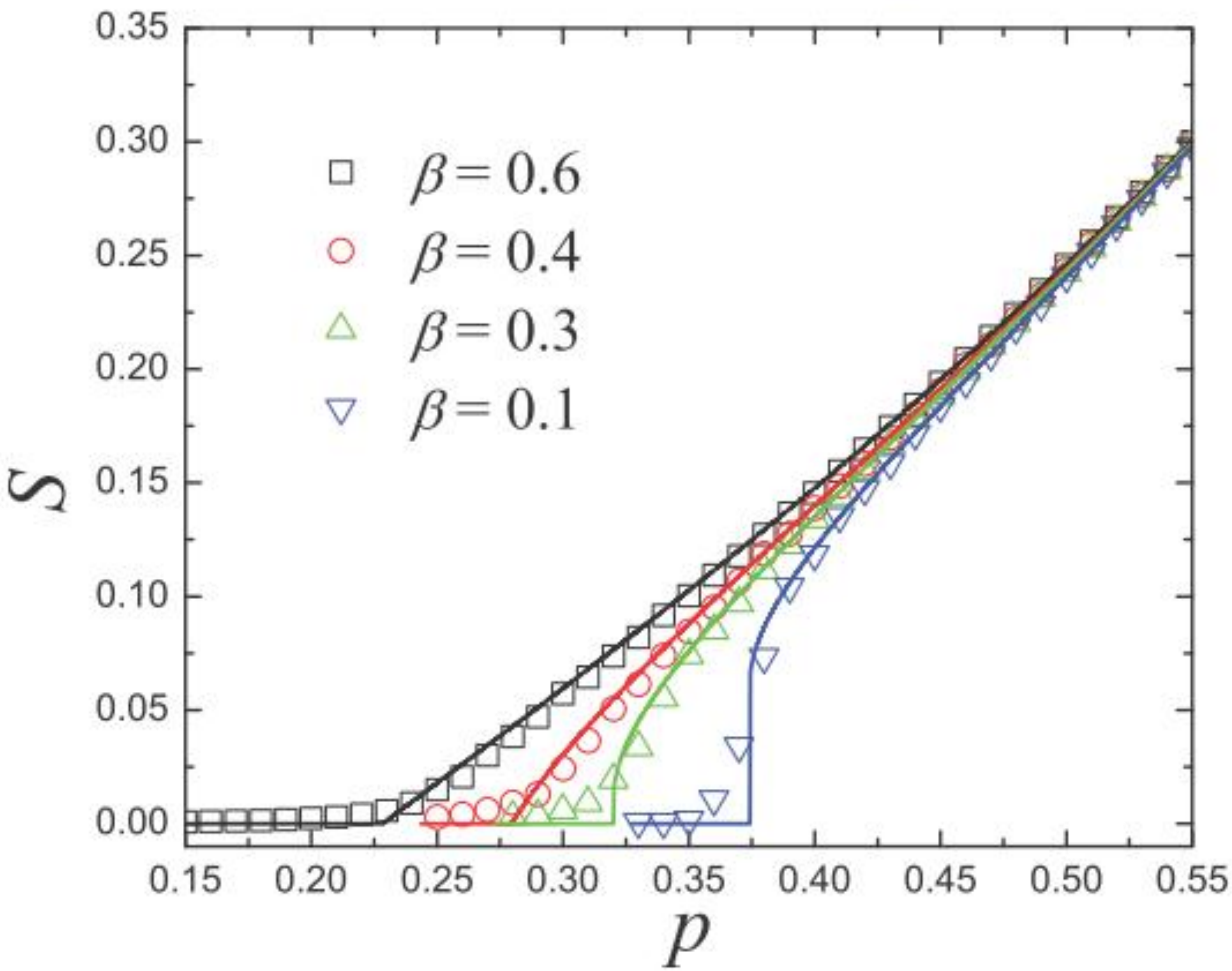
- The networks are not always vulnerable by making nodes interdependent
- The interdependent nodes are usually connected



single network

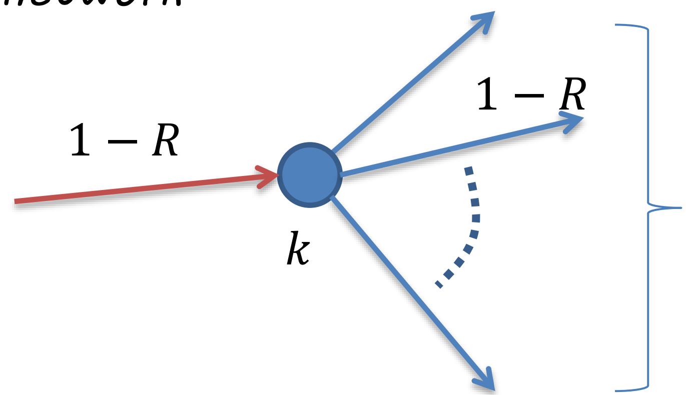
- *each node has exactly one dependence partner*
- *a fraction β of the dependence links overlaps with the connectivity links*

For a large fraction of overlapping, the system demonstrates a second order phase transition.



Analytical solution :

$1 - R$ is the probability that the node, arriving by following a randomly chosen link, does not belong to the giant component of the final network

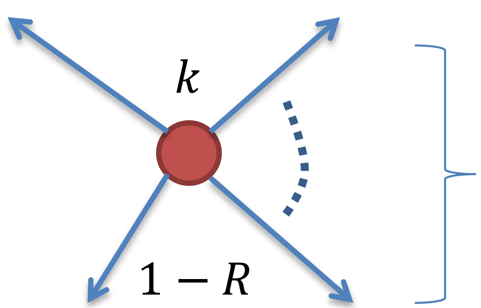


$(k-1)$

$$\sum_k \frac{P_k k}{\langle k \rangle}(1-R)^{k-1} = G_1(1-R)$$

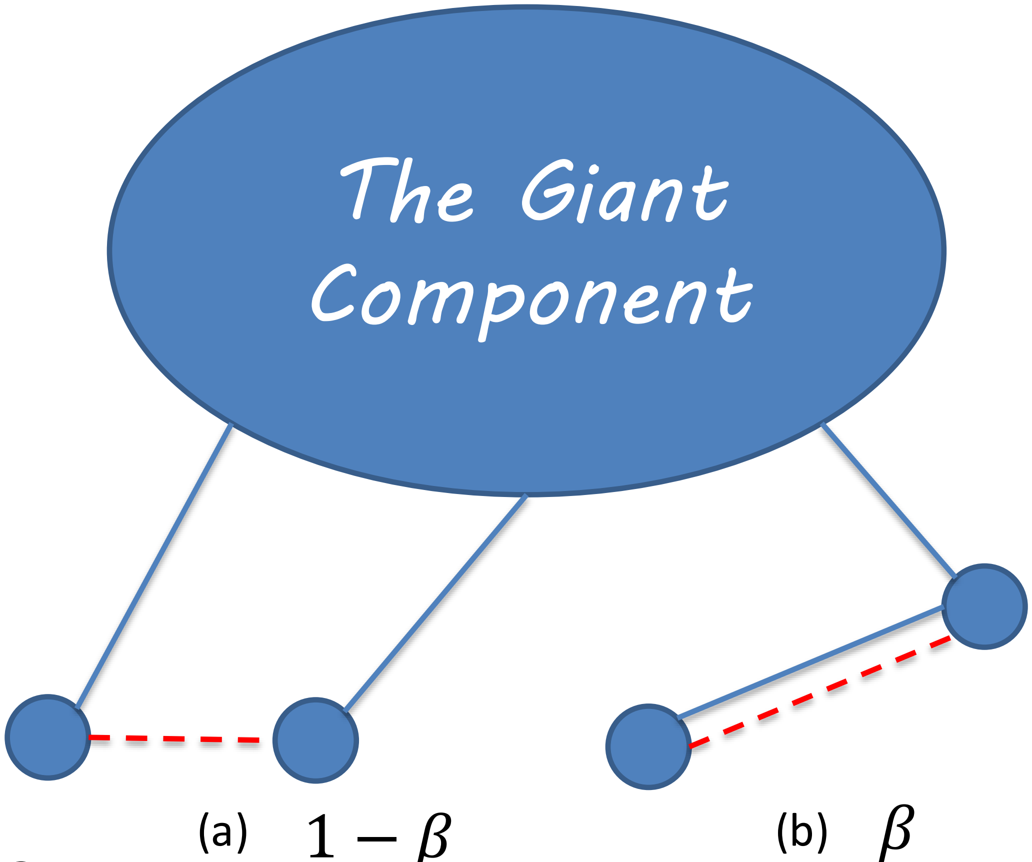
$1 - S$ is the probability that a randomly chosen node does not belong to the giant component of the final network

$$G_1(x) = G_0'(x)/G_0'(1)$$



k

$$\sum_k P_k(1-R)^k = G_0(1-R)$$



$$S = p^2 f$$

$$f = (1-\beta)[1 - G_0(1-R)]^2 + \beta\{1 - [G_1(1-R)]^2\}$$

Similarly, one can write down the equation for R:

$$R = p^2 g$$

$$g = (1-\beta)[1 - G_1(1-R)][1 - G_0(1-R)] + \beta\{1 - G_2(1-R)G_1(1-R)\}$$

Example:

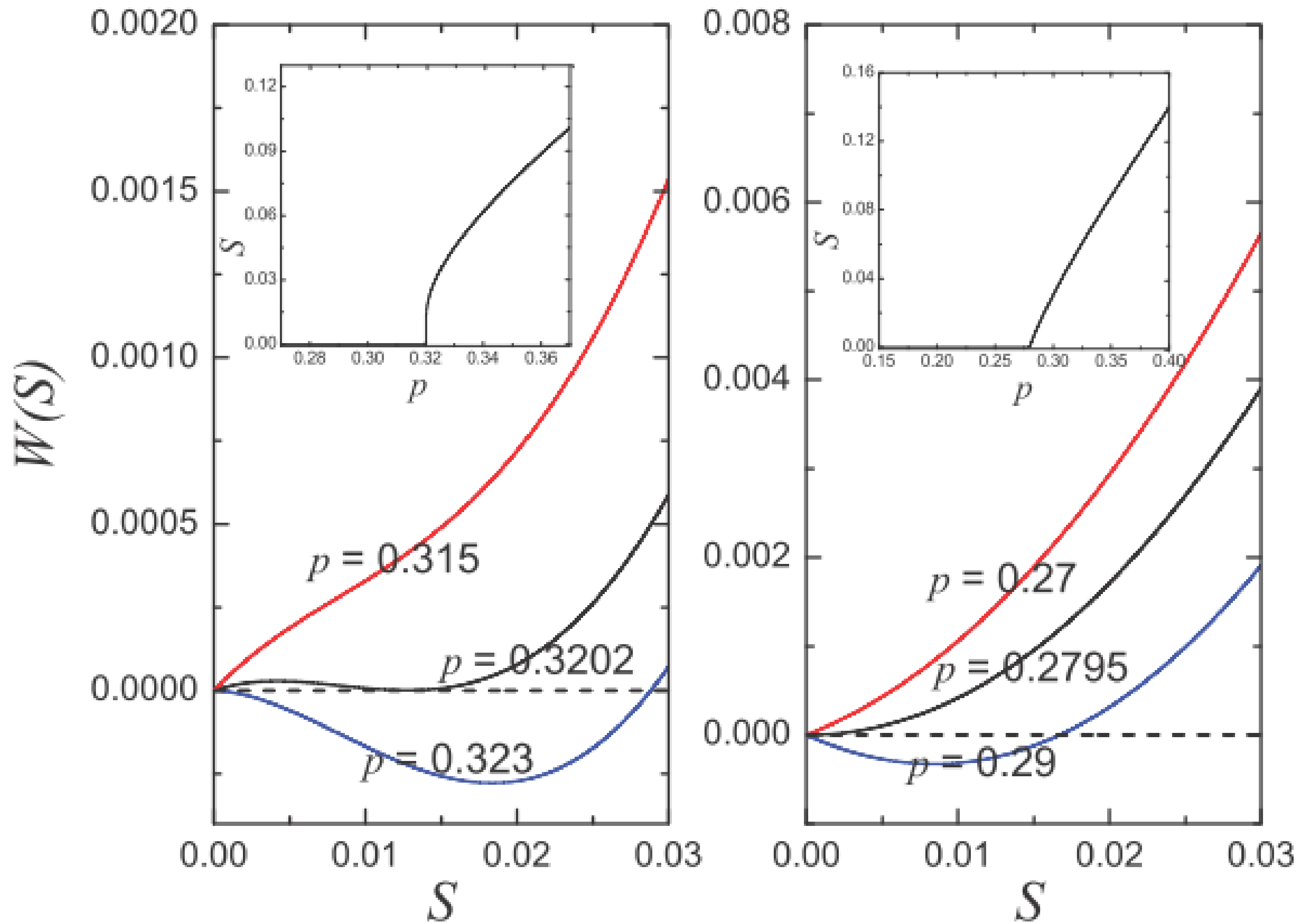
we consider a random network with degree distribution:

$$P_k = \frac{e^{-\langle k \rangle} \langle k \rangle^k}{k!}$$

Then, $G_0(x) = G_1(x) = G_2(x) = e^{-\langle k \rangle (1-x)}$

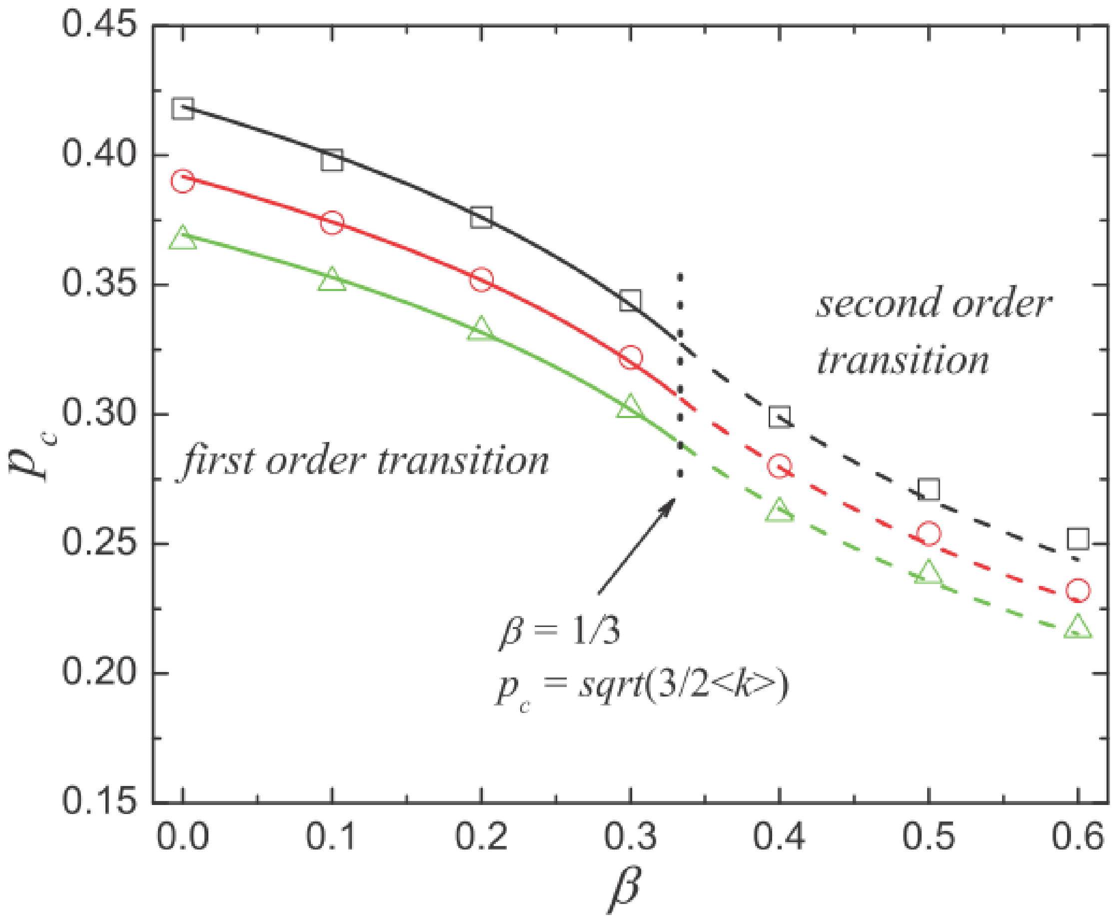
we have a simple self-consistent equation for the order parameter S:

$$S = p^2 \left\{ (1-\beta)\left(1 - e^{-\langle k \rangle S}\right)^2 + \beta\left(1 - e^{-2\langle k \rangle S}\right) \right\}$$



$\beta = 0.3$ first order $\qquad$ $\beta = 0.4$ second order

$$W(S) = S - p^2\left\{(1-\beta)\left(1-e^{-\langle k\rangle S}\right)^2 + \beta\left(1-e^{-2\langle k\rangle S}\right)\right\}$$



$$\begin{cases} \beta_c = \dfrac{1}{3} \\ p_c^* = \sqrt{\dfrac{3}{2\langle k\rangle}} \end{cases}$$

$$p_c^I = \left[2\langle k\rangle e^{-\langle k\rangle S}\left(1-\beta-(1-2\beta)e^{-\langle k\rangle S}\right)\right]^{-1/2}, \beta < \beta_c$$

$$p_c^{II} = \frac{1}{\sqrt{2\beta\langle k\rangle}}, \beta > \beta_c$$

5·Summary and outlook

- A high density of dependence links does not always make the percolation transition sharpened
- The correlation of dependence partners needs to be taken into account in the future work

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Thank you