Robustness of controllability for networks based on edge-attack

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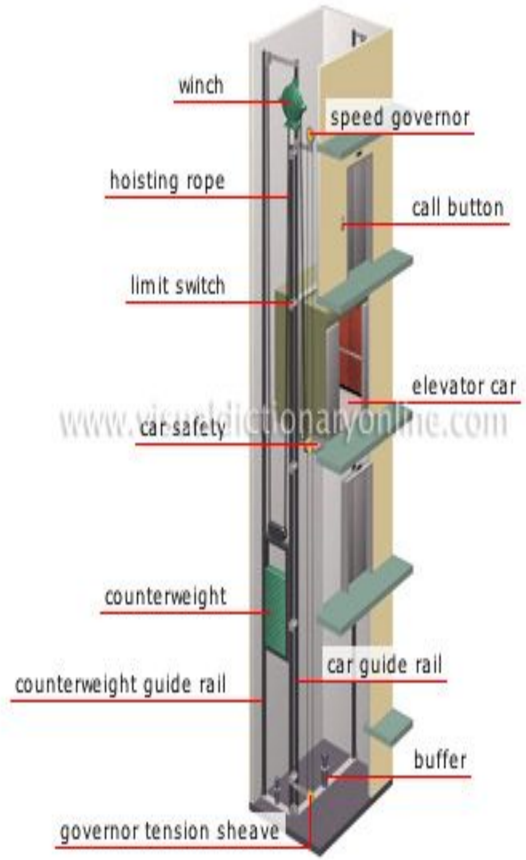
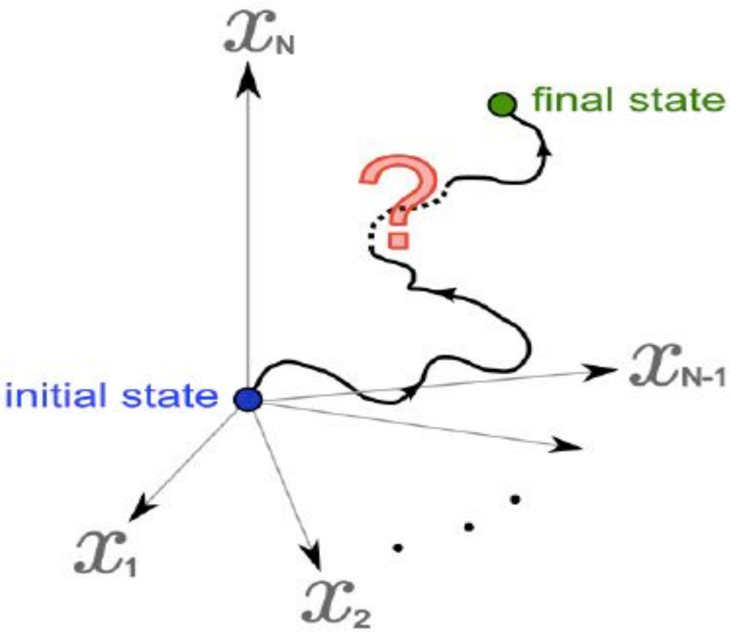


We need a few inputs to control a system.

Controllability

可控性：系统的所有状态向量都可以由输入来影响和控制

对于线性定常系统 $\dot{x}=Ax+Bu$，如果存在一个分段连续的输入 $u(t)$，能在 $[t_0,t_f]$ 有限时间间隔内，使得系统从某一初始状态 $x(t_0)$ 转移到指定的任一终端状态 $x(t_f)$，则称此状态是可控的。若系统的所有状态都是可控的，则称此系统是状态完全可控的，简称系统是可控的。



$$\frac{\mathrm{d}x(t)}{\mathrm{d}t}=Ax(t)+Bu(t)$$

线性定常系统： $$\frac{\mathrm{dx}(t)}{\mathrm{d}t}=A\mathbf{x}(t)+B\mathbf{u}(t)$$

$A=(a_{ij})_{N\times N}$ 系统矩阵 $u(t)$ 输入向量

$B=(b_{ij})_{N\times M}$ 输入矩阵

$x(t)=(x_1(t),x_2(t),\ldots,x_N(t))$ 状态向量

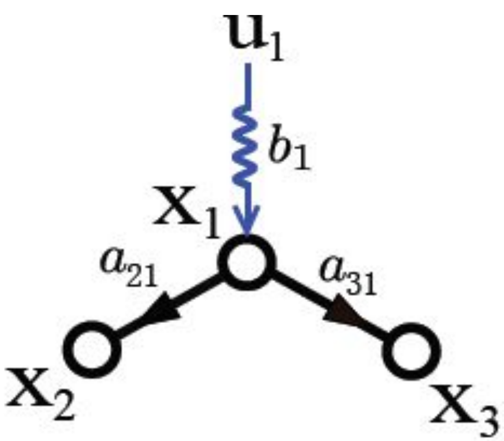
Kalman’s 可控秩判据：

$$C=(B,AB,A^2B,\ldots,A^{N-1}B)$$

$$\mathrm{rank}(C)=N$$

Structurally controllability

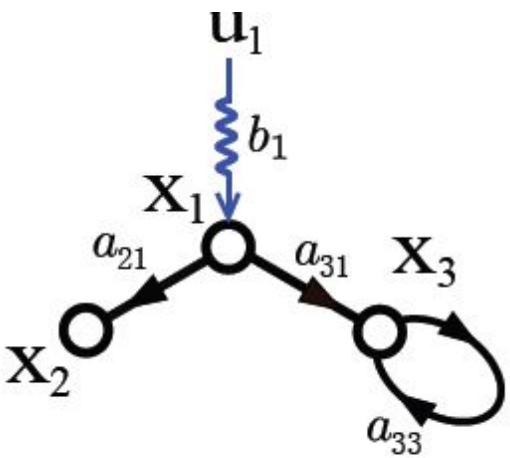
对于系统(A, B)，若选择任意非零权重，使系统满足秩判据



$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} u(t)$$

$$\mathbf{C} = [\mathbf{B}, \mathbf{A} \cdot \mathbf{B}, \mathbf{A}^2 \cdot \mathbf{B}] = b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & 0 \end{bmatrix}$$

$rank\ (C) \neq N$ 系统不可控



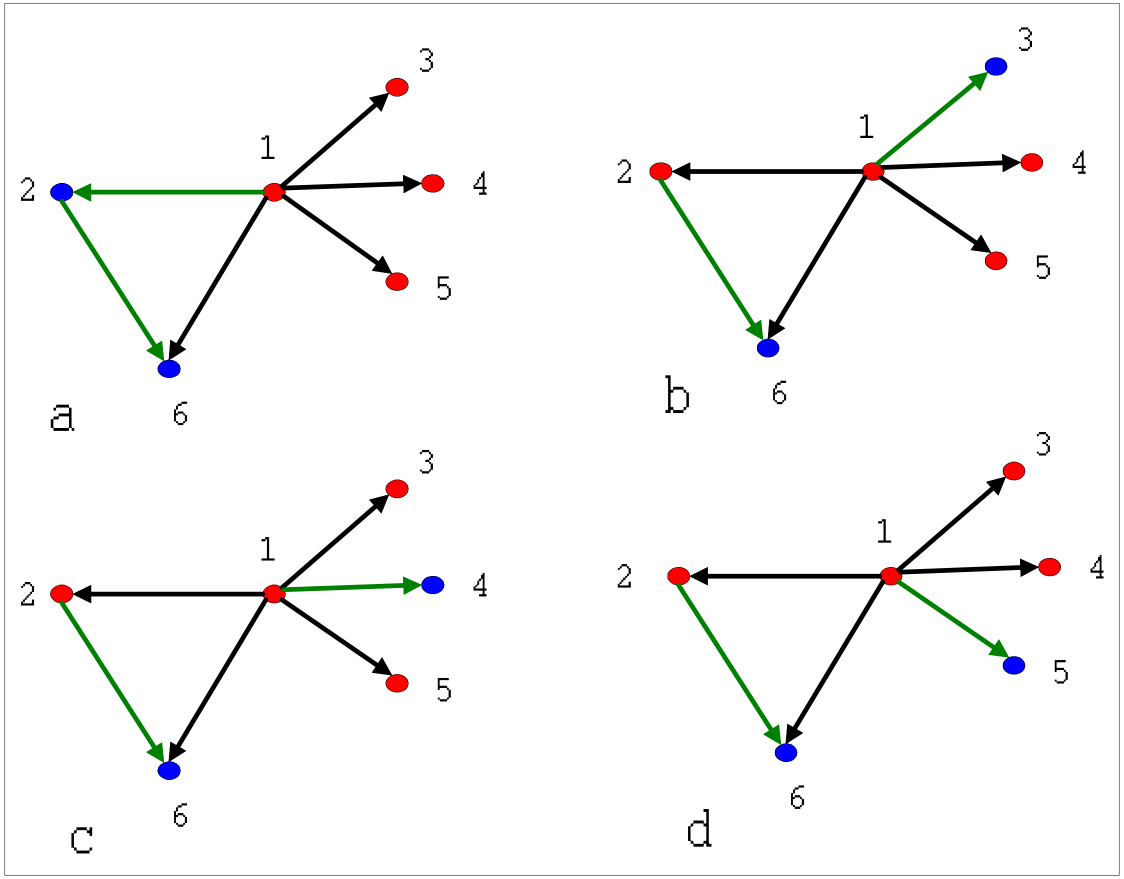
$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ a_{21} & 0 & 0 \\ a_{31} & 0 & a_{33} \end{bmatrix} \cdot \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix} u(t).$$

$$\mathbf{C} = [\mathbf{B}, \mathbf{A} \cdot \mathbf{B}, \mathbf{A}^2 \cdot \mathbf{B}] = b_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & a_{21} & 0 \\ 0 & a_{31} & a_{33}a_{31} \end{bmatrix}.$$

$$rank\,(C) = N$$

系统可控

使得系统成为连续化线性系统，推广至非线性系统



Maximum matching：边的最大集合，其中的边无公共起止点

Driver nodes（ ND ）：

若maximum matching中的边指向某节点，则该节点为matched，否则为unmatched,其中unmatched的节点即为驱动节点drive nodes

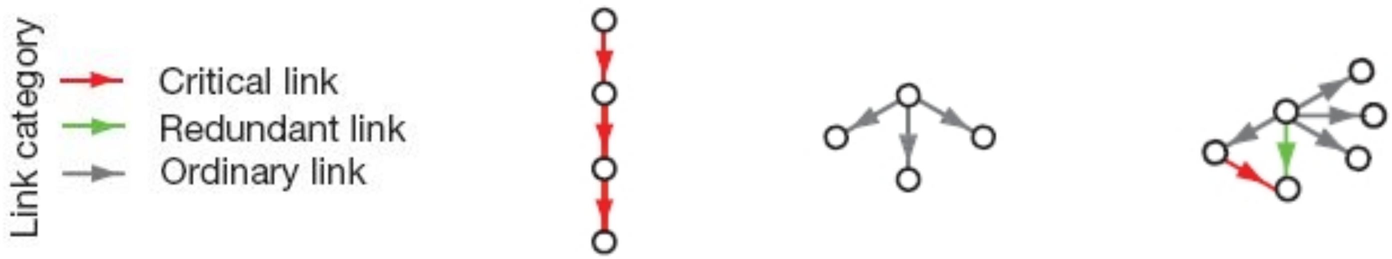
Liu et al. **Nature** 473, 167 (2011)

边的分类：

Critical: its removal needs to increase the number of driver nodes to maintain fully control;

Redundant: its removal cannot affect the current set of driver nodes;

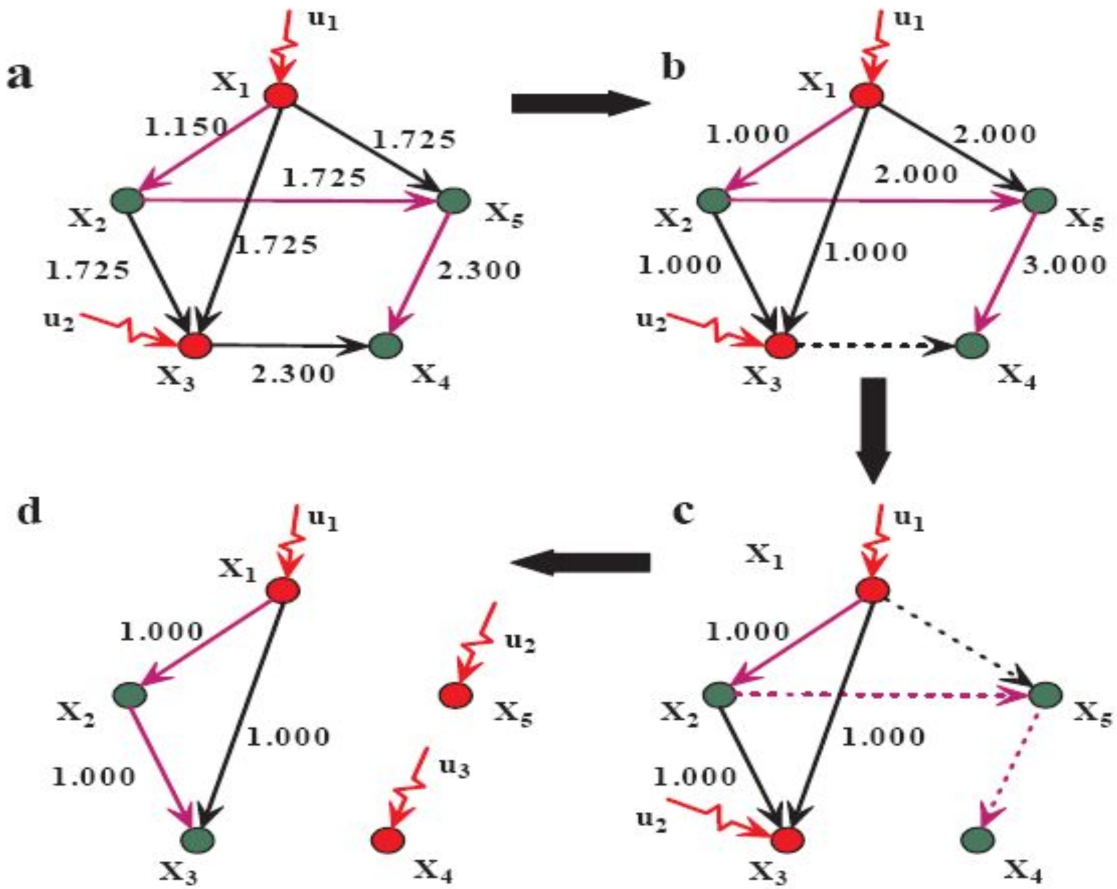
Ordinary: if it is neither critical nor redundant.



The cascading dynamics

$$H_{ij} = (1+\alpha)L_{ij}(0)$$

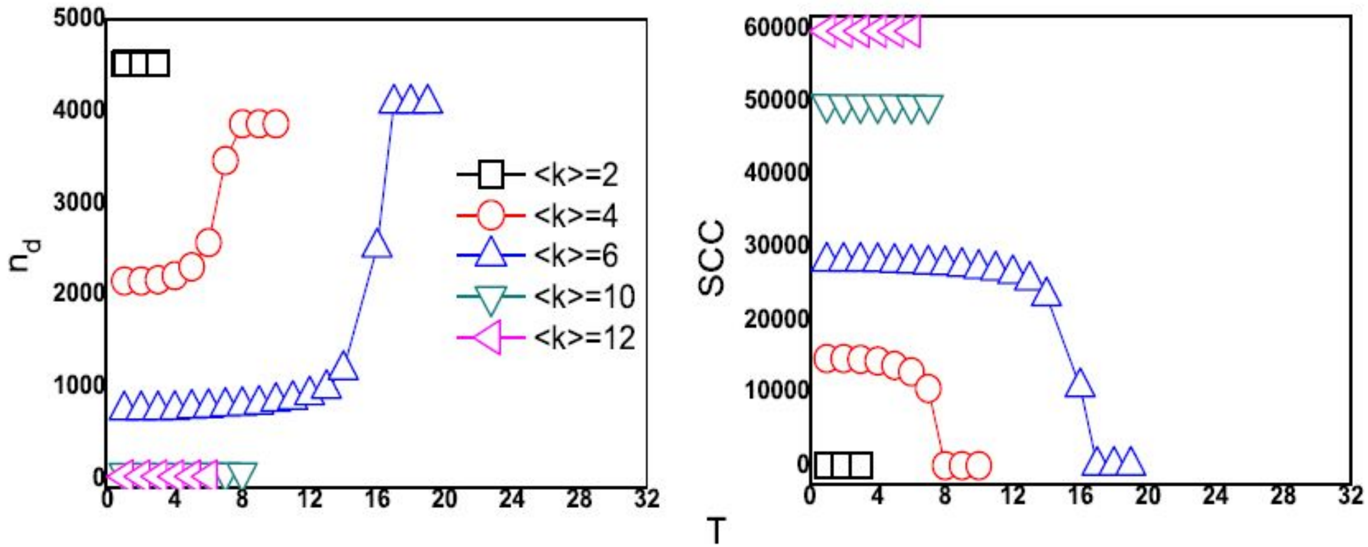
Motter et al. **PRE** 065102(2002)



Results

1. The highest-load edge attack

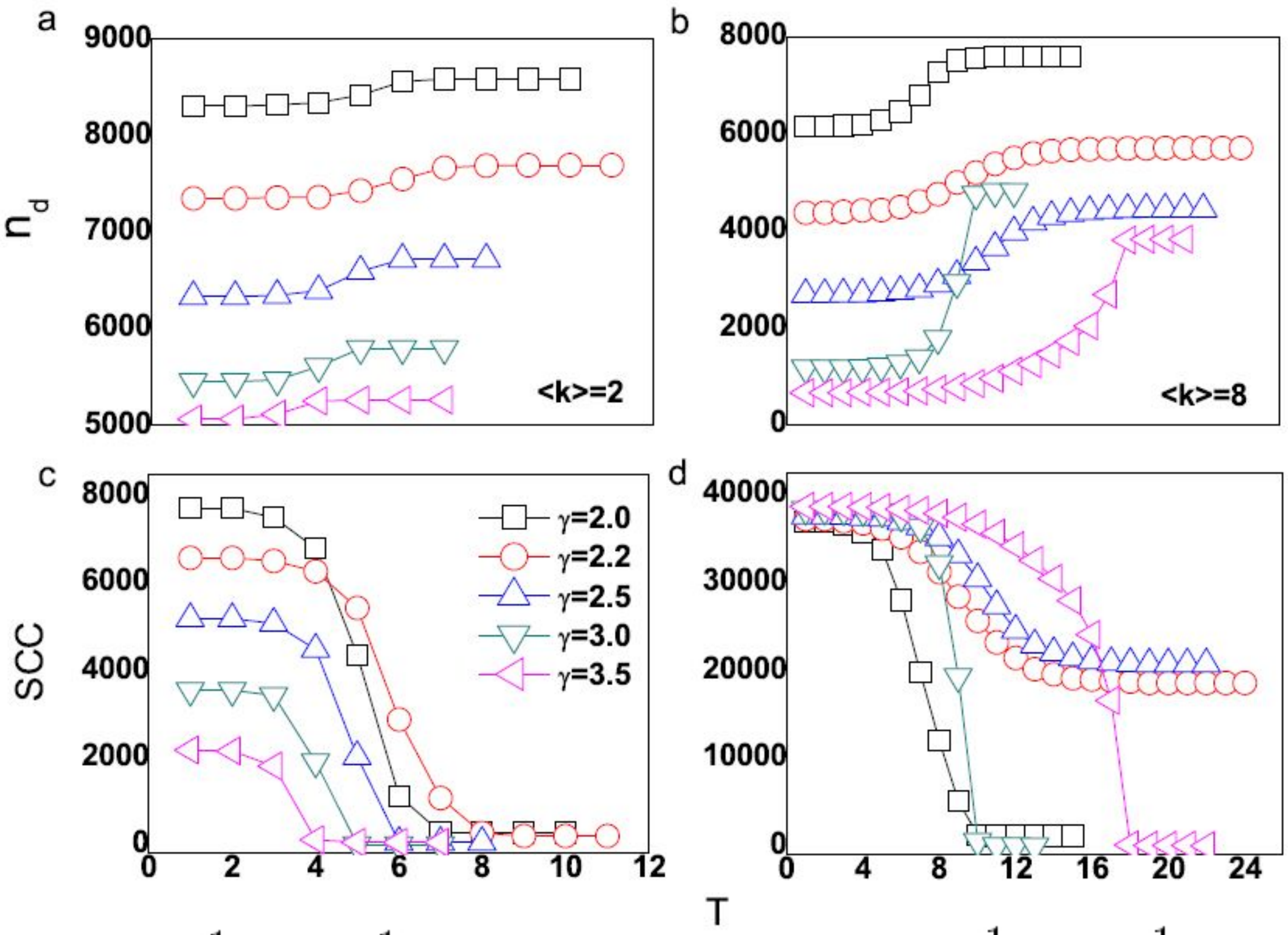
ER networks



$\alpha = 1.15 \quad N = 10000$

SCC: Strongly connected component

SF networks



$$n_d \approx exp[-\frac{1}{2}(1-\frac{1}{\gamma-1}) < k >]$$

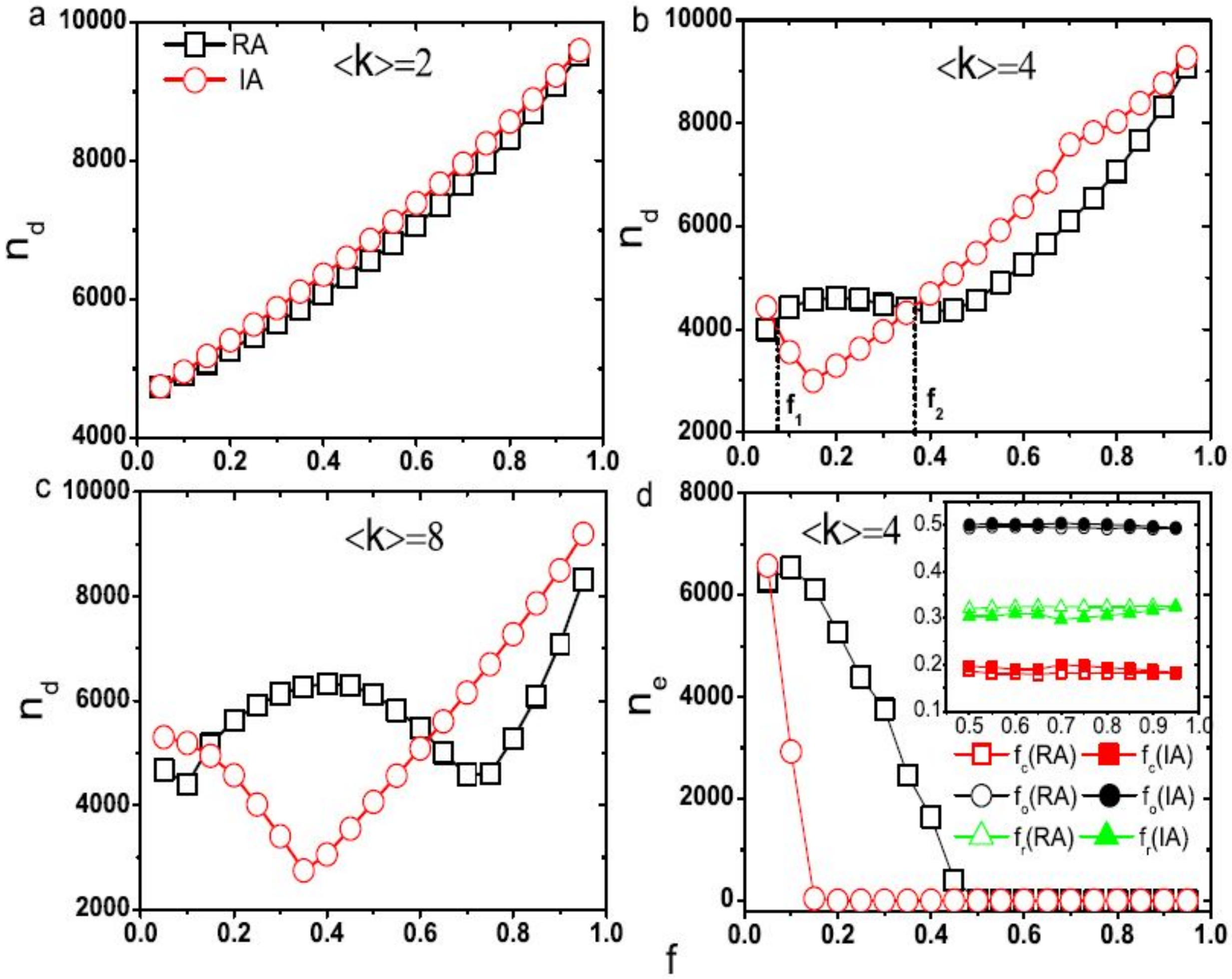
$$n_d^{'} \approx exp[-\frac{1}{2}(1-\frac{1}{\gamma-1}) < k^{'} >]$$

$$\Delta n_d = \frac{1}{2}(1-\frac{1}{\gamma-1})(k-k^{'})$$

2. A fraction of edges attack

RA: random attack   IA: intentional attack

ER networks



1. $f \le f_1$

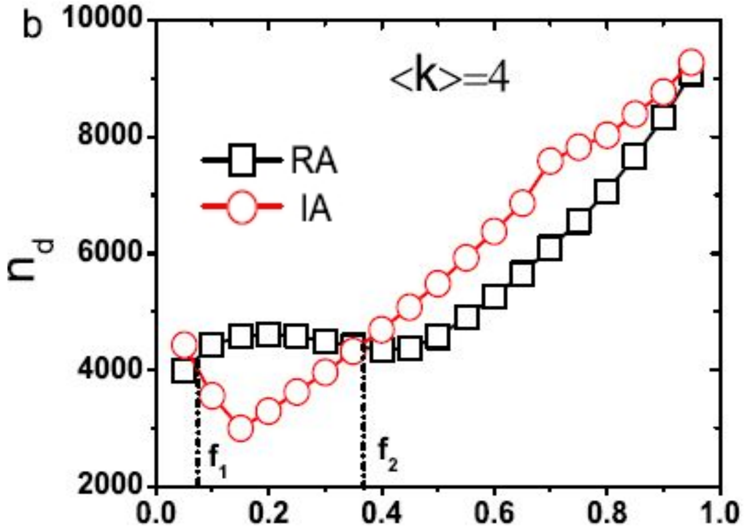
与highest-load edge attack类似

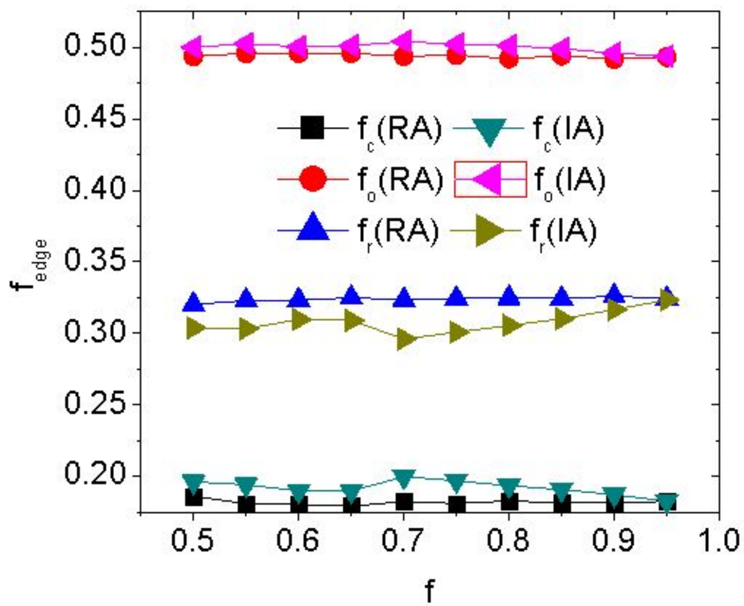
2. $f_1 < f \le f_2$

IA删边抑制了级联失效的传播

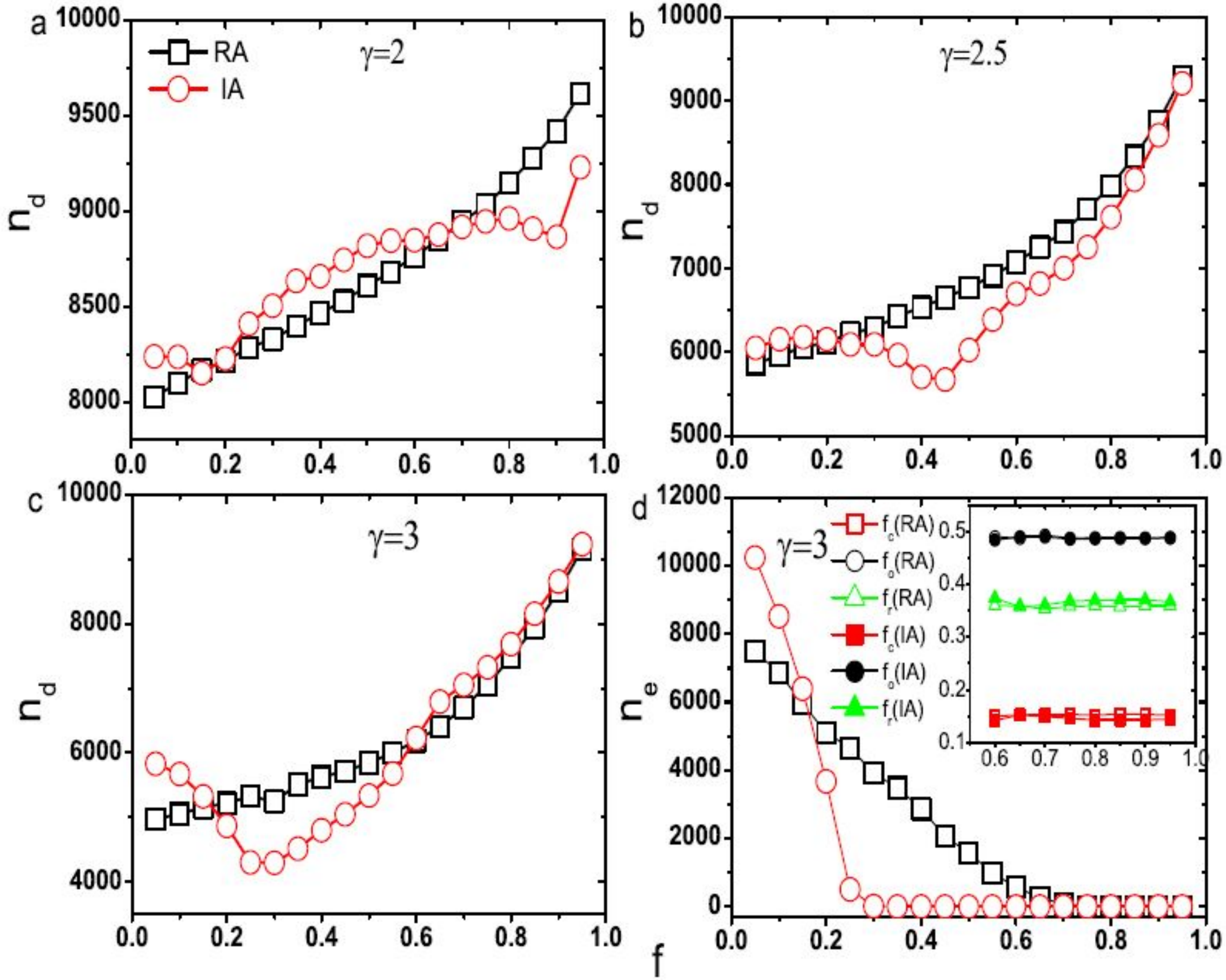
3. $f_2 < f \le 1$

IA 删边更容易删除网络中的critical边





SF networks



1. In the region of moderate f

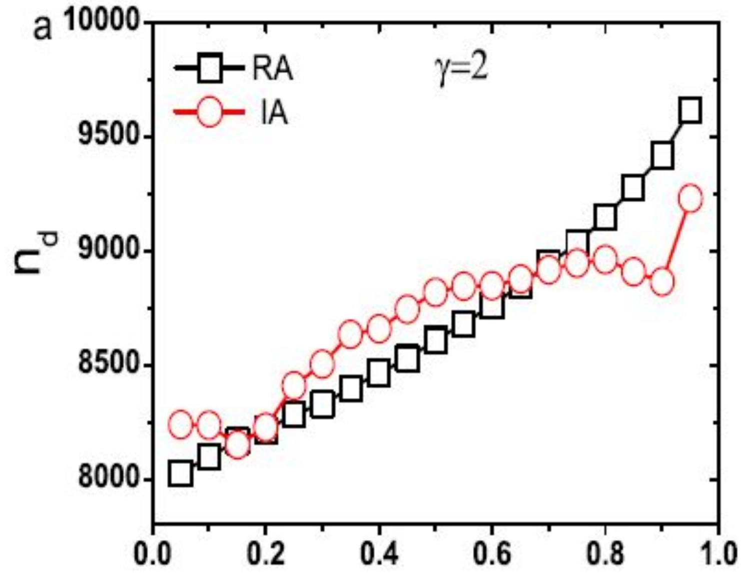
The intentional attack causes the network need more driver nodes than random attack

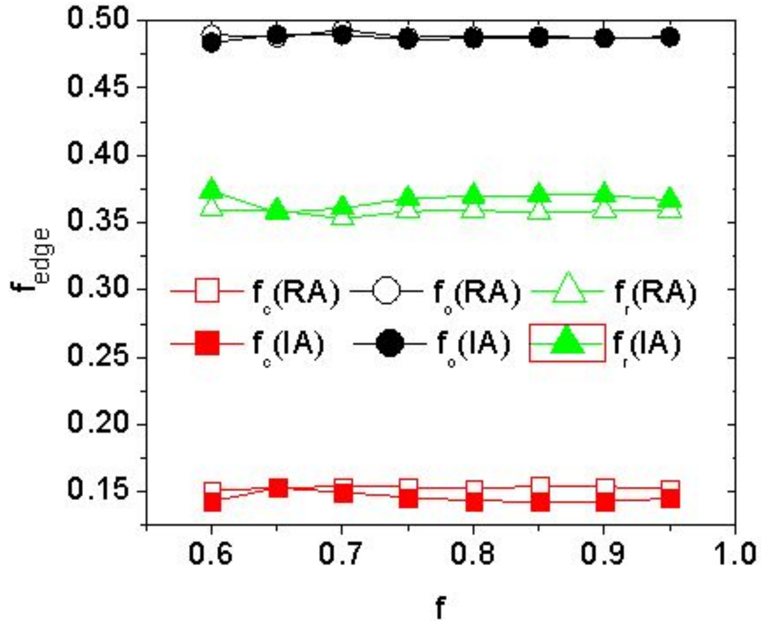
2. In the region of large f

The networks with small power exponent needs more driver nodes to maintain fully control under random attack than intentional attack.

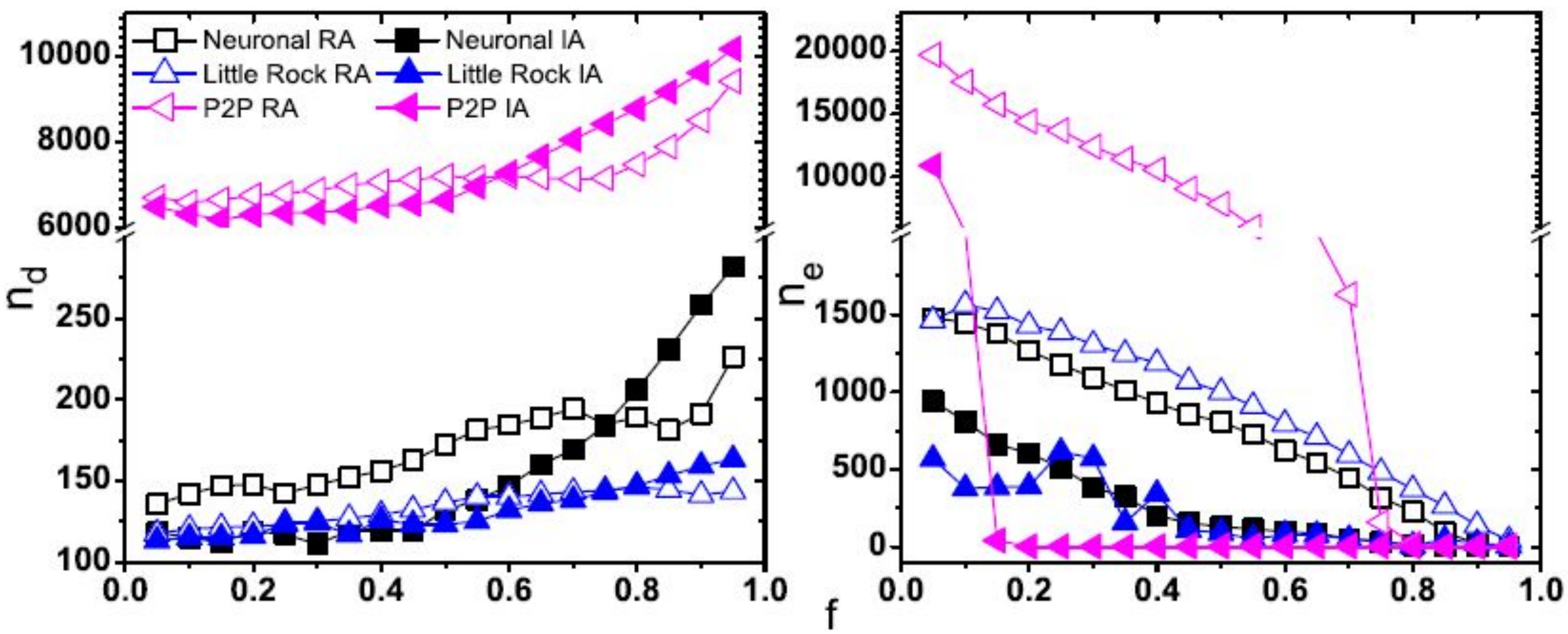
3. For larger γ

The difference of amount of driver nodes under two attacking strategies is not significant as larger f





Real systems



Thanks for your attention!