Catastrophes in Scale-Free Networks *

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An alternative model about cascading occurrences caused by perturbation is established to search the mechanism because catastrophes in networks occur. We investigate the avalanche dynamics of our model on two-dimensional Euclidean lattices and scale-free networks and find that the avalanche dynamic behaviour is sensitive to the topological structure of networks. The simulation results show that the catastrophes occur much more frequently in scale-free networks than those in Euclidean lattices, and the greatest catastrophe in scale-free networks is much more serious than that in Euclidean lattices. Furthermore, we have studied how to reduce the catastrophes' degree, and have schemed out an effective strategy, called the targeted safeguard strategy for scale-free networks.

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Many social, biological, and communication systems can be properly described as complex networks with vertices representing individuals or organizations and edges mimicking the interactions among them. Recently, the ubiquity of a power-law degree distribution in real-life networks has attracted much attention.[1−3] Examples of such networks (scale-free networks or SF networks for abbreviation) are numerous: these include the Internet, the World Wide Web, social networks of acquaintance or other relations between individuals, metabolic networks, integer networks, and food webs, etc.[4−9] The ultimate goal of the study of the topological structure of networks is to understand and explain the workings of systems built upon those networks: for instance, to understand how the topology of the World Wide Web affects Web surfing and search engines,[10] how the structure of social networks affects the spread of diseases, information, rumours or other things,[11] and so on.

The catastrophes in real-life networks can be seen everywhere, therefore it is not only of major theoretical interest, but also of great practical significance to understand the mechanism because those catastrophes occur. Intuitively, one may consider the breakdown of plentiful vertices or edges at the same time in networks to be the reason.[12] This is undoubtedly right as it is easy to imagine the damage of links or the failure of servers leading to a serious communication congestion, respectively. However, in most situations, the catastrophes surrounding us are unlike that. In this Letter, a man named "*perturbation*" is caught, who is indicted to be the causer in most catastrophes.

Bianconi and Marsili referred to an example about the catastrophe caused by perturbation.[13] The example they mentioned is routing tables in the internet, which can be considered as a dynamic communication network. In the beginning, a change (perturbation) in some router's table may inadvertently cause congestion at some node downstream. This may trigger several other changes in that local neighbourhood, as routers try to avoid the congested node. However, these changes may, in their turn, cause further congestion elsewhere, and the problem may expand even further, as a large avalanche (catastrophe), to a wider region. Similar phenomena may take place in various networks.

Another related example is the so-called fibre bundle system, which has been studied for many years in order to explain a variety of failure phenomena caused by cascades.[14−16] In the fibre bundle system, composed of N heterogeneous fibres put on a lattice, a fibre at the vth site is broken if the load σ_v is larger than the threshold value σ_v^{th} assigned to the fibre following a given probability distribution function. When the fibre is broken the load which was supported by the broken fibre is shared among intact fibres following a load sharing rule.

Bak, Tang and Wiesenfeld introduced a so-called sandpile model (BTW model) to explain such cascading occurrences on networks, which is considered as a prototypical theoretical model exhibiting the catastrophes (avalanche behaviour) caused by perturbation.[17] However, the BTW model is based on Euclidean lattices, which are very different from the reality for the real-life networks that have power-law degree distribution. In addition, the open boundary conditions make it difficult to directly extend the BTW model onto SF networks. Olami, Feder and Christensen established a model (OFC model) of earthquakes in nonconservative systems,[18] which

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may be more appropriate to mimic the catastrophes in SF networks than the BTW model since all real-life systems are nonconservative.

Recently, a few interesting and significant works about how the topological structure of networks affects self-organized criticality (SOC) based on the BTW model or the OFC model have been achieved. Lise *et al.* investigated the OFC model on annealed and quenched random networks,[19,20] Arcangelis and Herrmann studied the BTW model on small-world networks.[21] Motter and Lai studied the cascade-based attacks on SF networks and have found that a large-scale cascade may be triggered by removing a single key vertex.[22] Goh *et al.* investigated the avalanche dynamics on SF networks using the BTW model and obtained the exponent τ for the power-law avalanche size distribution and the dynamic exponent z.[23] Their work concentrated on the existence of SOC, thus they did not discuss whether the catastrophes occur more frequently in SF networks than in Euclidean lattices. In addition, since the threshold value of each vertex is assigned to be equal to its degree, one cannot be sure which (the power-law degree distribution, the power-law threshold height distribution or both) is the main reason that leads to the power-law distribution of avalanche size.

In this Letter, an alternative model is established to mimic the catastrophes occurring in networks. We have found that the catastrophes occur much more frequently in SF networks than those in Euclidean lattices and the greatest catastrophe in SF networks is much more serious than that in Euclidean lattices. Furthermore, we have studied how to reduce the catastrophes' degree, and have schemed out an effective safeguard-strategy for SF networks.

In our model, each vertex of the network is associated with a real variable F_x, which is initially taken to be 0 and can be considered as energy, tension, flux or some other things. At each time step, a perturbation δ is added to a randomly chosen vertex x, which means that the variable F_x increases by δ, where δ is randomly selected in the interval $(0, 1)$. If F_x reaches or exceeds the threshold value Z_x, then the vertex x becomes unstable and the $(1-\varepsilon)Z_x$ energies topple to its neighbouring nodes, with a small fraction ε of energies lost: $F_x \to F_x - Z_x$, and $F_y \to F_y + (1-\varepsilon)Z_x/d(x)$ for all vertices y adjacent to x, where $d(x)$ is the degree of vertex x that denotes the number of neighbouring vertices of x. The parameter ε controls the level of conservation of the dynamics and it takes values between 0 and 1, where $\varepsilon = 0$ corresponds to the conservative case. Here, to avoid the system being overloaded in the end, ε is always set to be larger than 0. If this toppling causes any of the adjacent vertices receiving energies to be unstable, subsequent toppling follows on those nodes in parallel until there is no unstable node left. This process defines an avalanche.

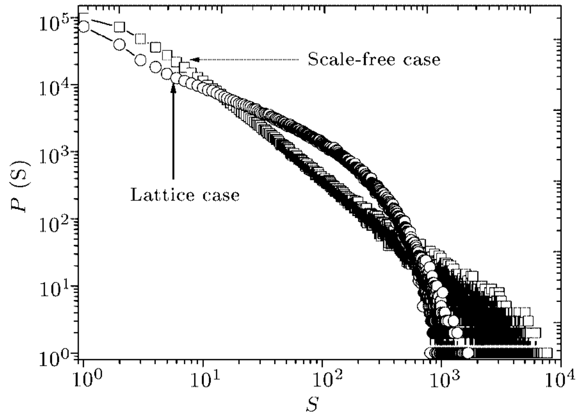

Fig. 1. Distribution of avalanche size with $\varepsilon = 0.01$, where $P(S)$ denotes the number of avalanches with give size S. Both the networks include 4900 vertices. The maximal avalanche size in the SF network is 8829 and the corresponding quantity in the Euclidean lattice is 1799. The data shown here are obtained by $10^6 + 10^5$ iterations excluding the initial 10^5 time steps.

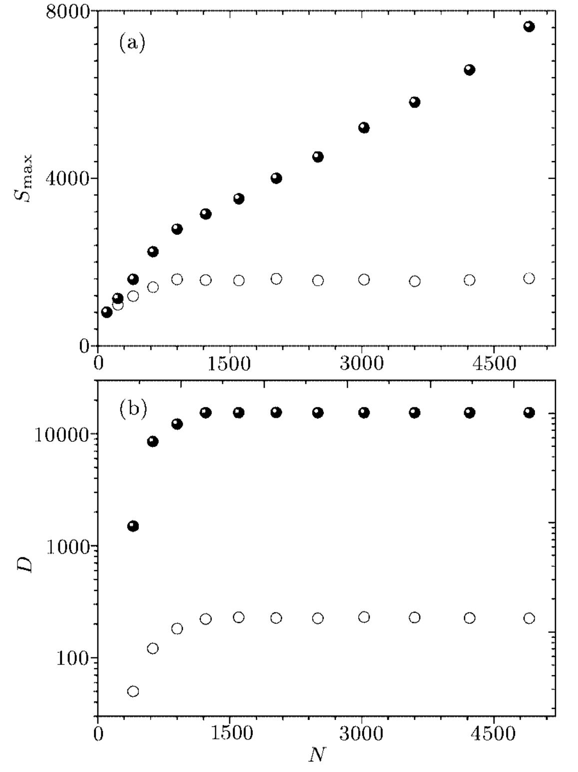

Fig. 2. Degree and frequency of catastrophes occurring in the two types of networks. For a fixed N, the data are the average of 10 independent experiments, the results for SF networks and Euclidean lattices are represented by closed and open circles, respectively. (a) The size of greatest catastrophes in SF networks is larger than that in Euclidean lattices, and the disparity becomes greater as the network-size increases. (b) The frequency of catastrophes occurring in SF networks is about 50 times higher than that in Euclidean lattices.

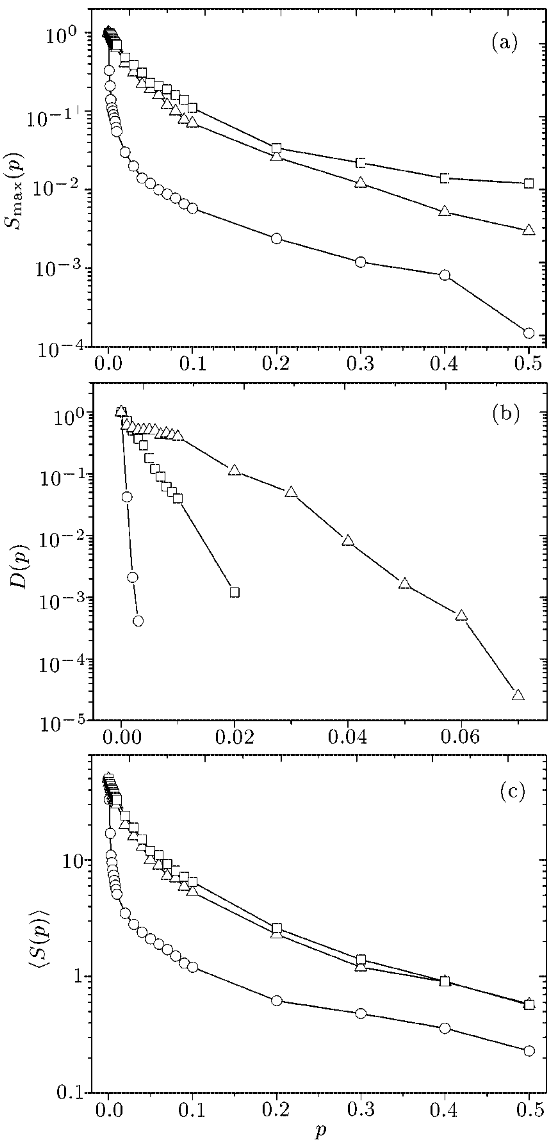

Fig. 3. Catastrophes in networks under vertex-protection with $N = 4900$ and $\langle d\rangle = 4$: (a) the maximal avalanche size $S_{\max}$, (b) the number of catastrophes D ($C = 1000$), and (c) the average avalanche size $\langle S\rangle$, versus protecting rate p. For a fixed p, the values shown here are the average over 10 independent experiments. The squares, triangles and circles denote the performance of RSS (TSS) in Euclidean lattice, RSS in SF network and TSS in SF network, respectively. The values of $S_{\max}$ and D are normalized by $S_{\max}(0)$ and $D(0)$, respectively.

In some networks such as the internet, the vertex with greater degree may have more throughput, thus Goh's designation for the threshold value of vertices is reasonable,[23] whilst in other networks (such as neural networks, social networks and so on), there is not any evidence that the individual having more neighbours is of greater endurance, thus it is worth studying the case that heterogeneous vertices are of the same threshold value. To make our description more specific, we use neural network as an example.[24,25] In the neural network, to each vertex, if a certain chemical matter exceeds a corresponding threshold, then it will be impressed to the neighbour vertices. Although the neural network is inhomogeneous, the threshold for a certain chemical matter is the same for each vertex. Therefore, in our model, the threshold values are assigned to be the same as $Z_x = Z = 1$ for all vertices x, which is different from Goh's values. It is notable that our designation for threshold value is helpful to clearly understand how the topology of networks affects the degree of catastrophe.

We are interested in the avalanche size S, which can be used to measure the degree of catastrophe and is defined as the number of toppling events in a given avalanche (we set $S = 0$ if there is no toppling event occurring). Figure 1 shows a typical result about the distribution of avalanche size. The Euclidean lattice mentioned in this paper is a two-dimensional square lattice under open boundary conditions, and the SF networks here are BA networks[26] with parameters $m_0 = m = 2$, thus both types of networks are of average degree $\langle d\rangle \simeq 4$. One can see that the distribution of avalanche size in SF network follows a straight line for more than three decades, which indicates that there is SOC in avalanche behaviour. However, the distribution of avalanche size in Euclidean lattices, which is a power-law curve in the left part followed by an approximately exponential truncation, is not similar to that in SF networks. Therefore, the dynamic behaviours in those two types of networks are different. The presence of SOC in the nonconservative OFC model has been controversial since the introduction of the model and it is still debated.[27–29] Since the main goal at present is to study the catastrophes occurring in networks, we would not give detailed simulation results and analysis on how the network structure affects the existence of SOC.

Getting to business, one can find that although the two networks are of the same network-size (i.e. the same number of vertices and edges), the maximal avalanche size $S_{\max}$ in SF networks is greater than that in Euclidean lattices, which means that the greatest catastrophe in SF network is much more serious than that in Euclidean lattices. A catastrophe here is defined as the avalanche with its size larger than an experiential lower bound C. Although we suppose $C = 1000$, we have checked that the phenomena are almost the same when a proper value of C, not too large or too small, is given. Therefore, our discussion does not depend upon the value of C. In SF networks, the number of avalanches with its size larger than C is 12291, and the corresponding quantity in Euclidean lattice is 231, which shows that the catastrophes occur much more frequently in SF network than those in Euclidean lattices. For the sake of reducing the error, more experiments have been performed. Figures 2(a) and 2(b) show the dependences of $S_{\max}$ and the number of catastrophes on the number of vertices N, which convincingly confirm the conclusions mentioned above.

Since there are not any effective methods to put an end to perturbations, it is worth studying how to reduce the catastrophes' degree. Here, for theoretical simplification, a safeguard-strategy is defined as a vertex set V_P with its elements protected and that will not topple (mathematically speaking, to protect a vertex x here means to set its threshold value Z_x as infinite). In this Letter, two safeguard-strategies are discussed, one is called the random safeguard strategy (RSS), and the other is called the targeted safeguard strategy (TSS). In the former, the vertices belonging to V_P are randomly selected; in the latter, the vertices with greater degree are chosen preferentially. Since almost all the vertices in Euclidean lattices are of the same degree, the RSS and TSS in Euclidean lattices are not discriminating.

We can always reduce the catastrophes' degree by protecting more vertices, but this could bring in economical and technical pressures. In order to roughly measure the economical and technical expenses, a parameter p called the protecting rate is defined as the proportion between the number of vertices protected and the total number of vertices: $p = |V_P|/N$. In Fig. 3, we report the simulation results about the different safeguard-strategies. From Fig. 3(a), one can find that the RSS in SF networks is slightly more effective than that in Euclidean lattices, and the TSS in SF networks is much more effective than RSS. For example, if we want to reduce the maximal avalanche size on SF networks to a tenth by using TSS, then to protect 0.5% vertices is enough; however, the protecting rate must be larger than 8% if we use RSS. According to the results shown in Fig. 3(b), if one wants to eliminate the catastrophes, at least 0.3%, 2% and 7% vertices should be protected by using TSS in SF networks, RSS in Euclidean lattices and RSS in SF networks, respectively. One can also find from Fig. 3(c) that TSS in SF networks is more effective than RSS: for example, if we set $p = 0.01$, then the average avalanche size will reduce to about 5 by using TSS, but the corresponding value is about 30 for RSS. Altogether, the above simulation results indicate that TSS is more effective than RSS for SF networks. The heterogeneity of vertices in SF networks is considered to be a possible reason for these results.

In summary, we have found that the avalanche dynamic behaviours are very sensitive to the topological structure of networks and the catastrophes are much more serious in SF networks than in Euclidean lattices. We have studied how to reduce the catastrophes' degree, and schemed an effective strategy (TSS) for SF networks, which may be of great significance in practice. However, there are many unanswered questions that puzzle us. How does the topological structure of networks affect the existence of SOC? Is the highly skewed degree distribution the key reason why the catastrophes in SF networks are much more serious than those in Euclidean lattices? Are there other safeguard-strategies more effective than TSS for SF networks?

References

[1] Albert R and Barabási A L 2002 *Rev. Mod. Phys.* **74** 47
[2] Dorogovtsev S N and Mendes J F F 2002 *Adv. Phys.* **51** 1079
[3] Newman M E J 2003 *SIAM Rev.* **45** 167
[4] Albert R, Jeong H and Barabási A L 1999 *Nature* **401** 130
[5] Jeong H et al 2000 *Nature* **407** 651
[6] Zhou T et al 2004 *Preprint* arXiv:cond-mat/0405258
[7] Chi L P et al 2003 *Chin. Phys. Lett.* **20** 1393
[8] Fan Y et al 2004 *Int. J. Mod. Phys.* B **18** 2505
[9] He Y, Zhu X and He D R 2004 *Int. J. Mod. Phys.* B **18** 2595
[10] Adamic L A et al 2001 *Phys. Rev.* E **64** 046135
[11] Yan G, Zhou T, Wang J, Fu Z Q and Wang B H 2005 *Chin. Phys. Lett.* **22** 510
[12] Albert R, Jeong H and Barabási A L 2000 *Nature* **406** 378
[13] Bianconi G and Marsili M 2003 *Preprint* arXiv:cond-mat/0312537
[14] Zapperi S et al 1997 *Phys. Rev. Lett.* **78** 1408
[15] Moreno Y et al 2000 *Phys. Rev. Lett.* **85** 2865
[16] Kim D H, Kim B J and Jeong H 2004 *Preprint* arXiv:cond-mat/0408580
[17] Bak P, Tang C and Wiesenfeld K 1987 *Phys. Rev. Lett.* **59** 381
[18] Olami Z, Feder H J S and Christensen K 1992 *Phys. Rev. Lett.* **68** 1244
[19] Lise S and Jensen H J 1996 *Phys. Rev. Lett.* **76** 2326
[20] Lise S and Paczuski M 2002 *Preprint* arXiv:cond-mat/0204491
[21] de Arcangelis L and Herrmann H J 2001 *Preprint* arXiv:cond-mat/0110239
[22] Motter A E and Lai Y C 2002 *Phys. Rev.* E **66** 065102
[23] Goh K I et al 2003 *Phys. Rev. Lett.* **91** 148701
[24] Eguíluz V M 2003 *Preprint* arXiv:cond-mat/0309092
[25] Kim B J 2004 *Phys. Rev. Lett.* **93** 168701
[26] Barabási A L and Albert R 1999 *Science* **286** 509
[27] Klein W and Rundle J 1993 *Phys. Rev. Lett.* **71** 1288
[28] Christensen K 1993 *Phys. Rev. Lett.* **71** 1289
[29] Carvalho J X and Prado C P C 2000 *Phys. Rev. Lett.* **84** 4006