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Role of Clustering Coefficient on Cooperation Dynamics in Homogeneous Networks *

WU Gang(吴刚), GAO Kun(高坤)**, YANG Han-Xin(杨涵新), WANG Bing-Hong(汪秉宏)***

Department of Modern Physics and Nonlinear Science Center, University of Science and Technology of China, Hefei 230026

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Based on previous works, we give further investigations on the Prisoners' Dilemma Game (PDG) on two different types of homogeneous networks, i.e., the homogeneous small-world network (HSWN) and the regular ring graph. We find that the so-called resonance-like character can occur on both the networks. Different from the viewpoint in previous publications, we think the small-world effect may be unnecessary to produce this character. Therefore, over these two types of networks, we suggest a common understanding in the viewpoint of clustering coefficient. Detailed simulation results can sustain our viewpoint quite well. Furthermore, we investigate the Snowdrift Game (SG) on the same networks. The difference between the outputs of the PDG and the SG can also sustain our viewpoint.

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Understanding the emergence of cooperative behaviour among selfish individuals is a fundamental problem in evolutionary biology.[1] Evolutionary game theory[2−7] has provided a powerful framework to characterize and investigate this problem. Two simple games have attracted much attention in theoretical and experimental studies: Prisoner's Dilemma Game (PDG)[8] and the Snowdrift Game (SG).[9] In the PDG, both players should, independently and simultaneously, decide whether to cooperate with (C) or to defect (D) the other. If they both choose C (or D), each will obtain a payoff of R (or P). If one of them chooses C while the other chooses D, the defector will obtain a maximum payoff of T, and the cooperator gets S. The rank of the payoff values satisfies $T > R > P > S$ with $2R > T + S$. As a result, in a single round of the PDG, defecting keeps superior all the way whatever choices the co-player makes, which makes cooperators unable to resist the invasions from defectors.[10] Such an unfavorable experimental work[11,12] has stimulate the adoption of other games,[13,14] including the SG, which is a viable and biologically interesting alternative. Actually, the ranks of P and S are exchanged in the SG, as $T > R > S > P$. According to the payoff ranks of the SG, the better choice depends on the choice of the co-player: to defect if the co-player cooperates, or to cooperate if it defects. As a result, evolution carried out in a well-mixed population leads to an equilibrium frequency of cooperators given by $1-r$, with $0 \le r \le 1$ being the cost-to-benefit ratio of mutual cooperation.

Furthermore, instead of well-mixed population, complex networks have given a useful spatial structure for the evolutionary games. Supposing that individuals only interact with their immediate neighbours in a social network, several studies[4−7,15−24] have reported the asymptotic survival of cooperation on different types of networks. In fact, network structures and spatial characters have essential influences to the cooperative behaviours. Notably, cooperation even dominates over defection such as in the scale-free networks, in which the degree distribution follows power-law. Moreover, Tang *et al.* have discovered the influence of averaged connectivity on the cooperative behaviours.[25] More recently, Santos *et al.* introduced a homogeneous small-world networks (called HSWN for short), which exhibits a homogeneous connectivity distribution in order to make sure how the pure "small-world" effect influences on the cooperative behaviours without any associated heterogeneity.[26] Furthermore, Ren *et al.* have found out that a resonance-like fashion emerges on the HSWN for the PDG,[27] which is considered as the result of small-world effect and the topological randomness.

In this Letter, considering the previous works in Refs. [26,27], we further carry out the PDG and the SG on both the regular ring graph and the HSWN. It is interesting that similar resonance-like phenomena arise on both the two networks for the PDG. As the small-world effect and topological randomness does not exist in the regular ring graph, they are not necessary ingredients inducing the resonance-like phenomena. Therefore, it is needed to find out a common understanding over these two homogeneous networks. As an attempt, the clustering coefficient is laid out to explain these phenomena in the this study.

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** Email: kgao@mail.ustc.edu.cn

*** Email: bhwang@ustc.edu.cn

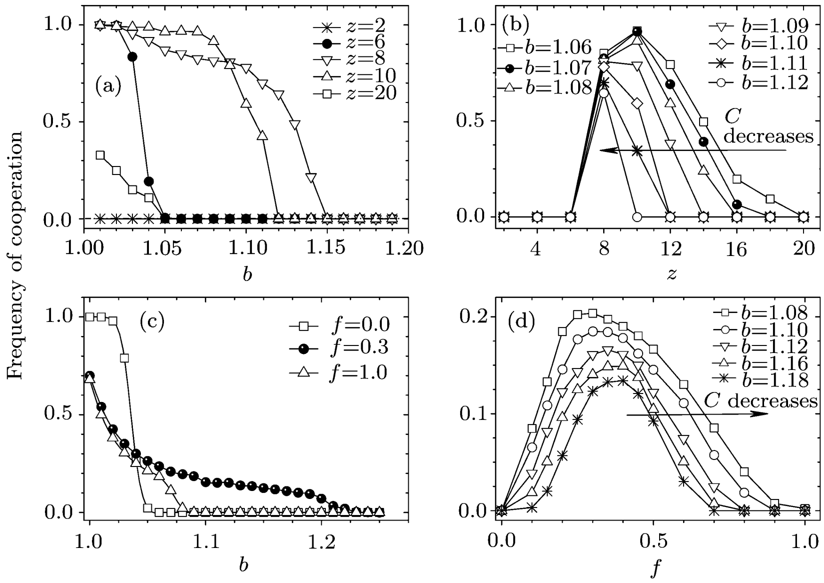

Fig. 1. Frequency of cooperation on the regular ring graphs and the HSWN. Results shown as a function of the payoff parameter b in (a) and (c), of the connectivity Z in (b), and of the fraction of swapped edges f in (d), respectively. Similar phenomena appear in the two different networks. The baseline $p_c = 0$ in (a) corresponds to a well-mixed population (complete graph). The arrows in (b) and (d) denote the directions for the clustering coefficient C decreasing. The network size is $N = 4000$ for the regular ring graphs and $N = 1000$ for the HSWN.

We firstly construct the networks, i.e. the regular ring graph and the HSWN. For the regular ring graph, nodes are arranged on a circle, with each node connected to the Z most-neighbouring nodes. On the other hand, the HSWN, in contrast with the Watts–Strogatz (WS) model, has small-world effect together with keeping the connectivity of each player unchanged. Starting from a undirected regular ring graph, a two-step circular procedure is introduced: (1) choose two different edges randomly, which have not been used by yet in step (2), and (2) randomly swap the ends of the two edges. Of course, duplicate connections and disconnected graphs are prohibited. A dimensionless parameter f, defined as the ratio of swapped edges to the total edges in the network, is introduced to denote the topological randomness.[26]

After the networks are constructed, each node is occupied by a player. Each player can be either a cooperator (C) or a defector (D). All pairs of connected individuals play the game simultaneously and gain benefits according to the payoff matrix of the game. Following previous works,[15,28,29] we adopt the rescaled version of payoff parameters as $R = 1$, $P = S = 0$, $T = b$ for the PDG, and $R = 1$, $P = 0$, $S = 1 - r$, $T = 1 + r$ for the SG. Thus there is only one variable payoff parameter in each game. The payoff matrixes for the PDG and the SG are

PDG

	C	**D**
C	1	0
D	b	0

SG

	C	**D**
C	1	$1-r$
D	$1+r$	0

.

Players interact only with their immediate neighbours. The total payoff of each player is calculated by summing the payoffs over all its interactions at each time step. At the next generation, each player, for example x, randomly selects one of its neighbours such as y. Supposing E_x and E_y are the total payoffs of player x and y in the previous round, player x will adopt player y's strategy with probability

$$W = \frac{1}{1 + \exp[(E_x - E_y)/K]}, \tag{1}$$

where K characterizes the noise introduced to permit irrational choices. For $K = 0$, player x will choose to adopt player y's strategy deterministically if $E_y > E_x$, or not if $E_y < E_x$. The finite value of K characterizes the range of payoff difference within which the irrational decision can typically appear. In the present study, we fix $K = 0.08$. Moreover, Eq. (1) is called the "Fermi-like distribution", which is frequently used for game models. It should be noted that the simulation results in this study do not rely on the detailed form of Eq. (1).

One of the key quantities for characterizing the cooperative behaviour is the frequency of cooperators p_c, which is defined as the fraction of cooperators in the whole population. In all simulations in this study, p_c is obtained by averaging over the last 5000 time steps out of entire 15000 time steps and each data point results from 10 different network realizations. Initially, strategies C and D are uniformly distributed among all players. In Fig. 1, we plot the frequency of cooperation for the PDG on both the regular ring graphs with different connectivities Z, and the HSWN with different values of f. The top panels show the results on regular ring graphs. In Fig. 1(a), one can find that when $Z = 2$, cooperators can not survive when $b > 1.0$. With the increase of Z, the cooperation level can be substantially promoted for large b. However, when $Z = N - 1$ (N denotes the network size), the frequency of cooperation falls to the baseline $p_c = 0$. Therefore, there should be a maximum cooperation level with an intermediate value of Z if we plot the cooperation level p_c versus the topological connectivity Z, which is shown clearly in Fig. 1(b). This resonance-like phenomenon has been reported on the HSWN in Ref. [27]. In comparison, the corresponding results on the HSWN are shown in Figs. 1(c) and 1(d). Furthermore, it should be also noted that the optimal value of Z can be influenced by the values of b. In Fig. 1(b), one can find that the peaks corresponding to the optimal topological connectivities move leftward as b increases.

In Ref. [27], provided that the networks are homogeneous, the resonance-like character as shown in Figs. 1(b) and 1(d) are explained as a result of the small-world effect. However, as shown above, this resonance-like character phenomenon can occur on the regular ring graph, which does not have the characters of small-world networks. Therefore, the small-world effect may not be a necessary ingredient for this character. It is needed to find out a common explanation over these two types of homogeneous networks. In our viewpoint, the analysis through the clustering coefficient may provide a valuable way to understand this problem on the same assumption that the network is homogeneous (Ref. [27] has pointed out that the heterogeneity of network can eliminate the resonance-like character). The clustering coefficient is defined as the average fraction of pairs of neighbours of a node that are also neighbours of each other.[30,31] Through a simple calculation, we could know that a regular ring graph with a connectivity $Z \ll N$ has a clustering coefficient

$$C(Z) = \frac{3(Z-2)}{4(Z-1)},$$

which is an increasing function of Z. On the other hand, Fig. 2(a) shows the clustering coefficients $C(f)$ and average path lengths $L(f)$ relative to the values for regular networks $C(0)$ and $L(0)$, as functions of the topological randomness f for the HSWN. It is obvious that the clustering coefficient C is a descending function of f. The arrows in Figs. 1(b) and 1(d) denote the directions of decreasing C, in which both the optimal values of Z and f move along when b increases.

Thus we can give a common explanation to the resonance-like phenomenon over these two types of networks through the viewpoint of clustering coefficient. When the clustering coefficient is small, the interactions between cooperators inside a cooperative cluster is few so that the payoffs for cooperators are inappreciable. It makes the defectors easy to penetrate. For example, for the case $Z = 2$ (in which $C = 0$) for the regular graph, only one defector in the system can make all cooperators extinct. On the other hand, when C is extremely large, a defector located on the edge of a cooperator cluster can exploit more cooperators and gain a quite large benefit, which makes it have stronger ability to penetrate the cooperator cluster. In consequence, more defectors appear in the system. Therefore, middle values of the cluster coefficient are most suitable for cooperation. On the other hand, with the increase of b, defectors will benefit more from cooperators. In this case, smaller clustering coefficient is needed to inhibit the defectors' benefit and to weaken their ability of penetrating. As a result, the optimal value of clustering coefficient decreases.

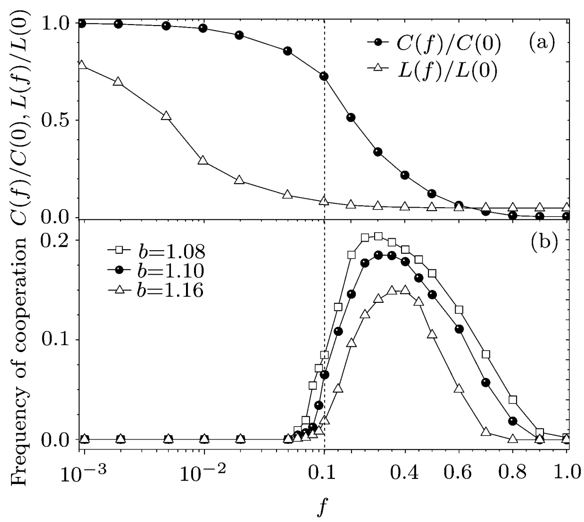

Fig. 2. Clustering coefficients $C(f)$ and average path lengths $L(f)$ relative to the values for regular networks $C(0)$ and $L(0)$, shown as a function of f for the HSWN. (b) Cooperative frequency p_c shown as a function of f. The left panels (from left edge to the dashed line) show in logarithm scales that when $f \leq 0.05$, L decreases typically while C decreases quite slowly, and p_c keeps zero. On the contrary, when $f > 0.05$, C decreases typically while L decreases slowly, and p_c varies non-monotonically. The network parameters are $N = 1000$, $Z = 6$. Each date point results from an average over 10 individual samples.

In more detail, we plot the frequency of cooperation on the HSWN over small topological randomness f in Fig. 2(b). Reference [26] remarks that the short average distance promotes the spreading of cooperators. As a result, the cooperative behaviour is promoted correspondingly, when f is not too large. However, Figs. 2(a) and 2(b) show that when $f \leq 0.05$, the average distance of the network is shortened typically with increasing f, but the clustering coefficient C and the cooperative frequency p_c are kept nearly unchanged. Here $f = 0.05$ marks the onset of a more rapid change of cooperative frequency with the rapid change of C. Therefore, based on this fact, we believe that the clustering coefficient has much more important influence on the cooperative behaviour than the average distance.

Next, we study the SG on the HSWN as an example and find only trivial behaviours, as shown in Fig. 3. The main difference between the PDG and the SG is caused by different characters of cluster between them. Reference [28] has pointed out that when defectors has typically more benefits than cooperators, globular clusters of cooperators will form in the PDG, which makes cooperators enough protection to persist at a small frequency. While in the SG, the clusters become more finger-like, or dendritic, which are easy to be penetrated and eliminated. Although the SG favors cooperation better than the PDG, it could not sustain globular clusters of cooperators which is important for the resonance-like phenomenon. This has given an assistant proof that the cluster characters is essential to the cooperative behaviours and the resonance-like phenomenon. Thus to understand in the view of clustering coefficient is acceptable and reliable.

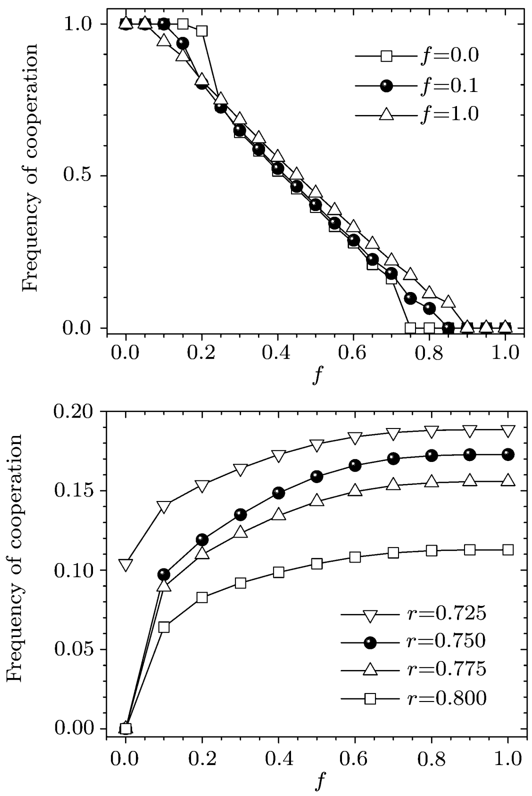

Fig. 3. (a) Frequency of cooperators p_c over the payoff parameter r for the SG on the HSWN for different f; (b) p_c as a function of f for different r. The network parameters are $N = 1000$ and $Z = 10$.

In summary, we have mainly given further investigations on the Prisoners' Dilemma Game on the homogeneous small-world networks (HSWN) and then developed it to the regular ring graphs. We find that the so-called resonance-like phenomenon can occur on both the types of networks, rather than occurs particularly on the former. In contrast to the previous work by Ren *et al.*, we suggest a different viewpoint to understand the resonance-like character based on the clustering coefficient. This viewpoint of clustering coefficient can explain the phenomena realistically and is common to both the homogeneous networks. Detailed character of cooperative behaviours can indicate that the clustering coefficient plays a more important role in the emergence of the resonance-like character than the small-world effect. The different outcomes of the SG to the PDG has also given assistant proofs to our viewpoint.

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