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Evolutionary snowdrift game on heterogeneous Newman–Watts small-world network*

Yang Han-Xin(杨涵新)†, Gao Kun(高 坤),
Han Xiao-Pu(韩筱璞), and Wang Bing-Hong(汪秉宏)‡

Department of Modern Physics and Nonlinear Science Center, University of Science and Technology of China, Hefei 230026, *China*

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We study the evolutionary snowdrift game in a heterogeneous Newman–Watts small-world network. The heterogeneity of the network is controlled by the number of hubs. It is found that the moderate heterogeneity of the network can promote the cooperation best. Besides, we study how the hubs affect the evolution of cooperative behaviours of the heterogeneous Newman–Watts small-world network. Simulation results show that both the initial states of hubs and the connections between hubs can play an important role. Our work gives a further insight into the effect of hubs on the heterogeneous networks.

Keywords: complex networks, snowdrift game, cooperation, heterogeneous Newman–Watts small-world network

PACC: 0250, 0175, 0270

1. Introduction

Cooperative behaviours are ubiquitous in many biological, social and economic systems.[1] Yet, understanding the emergence and persistence of cooperation remains a challenge to many natural and social scientists.[2] So far, game theory has provided a common mathematical framework to characterize and investigate the evolution of cooperation.[3−6] The Snowdrift game (SG),[7] as a general model, is often used in this field. In the original SG, two players can determine whether to cooperate or to defect. They both receive the payoff R upon mutual cooperation and the payoff P upon mutual defection. If one defects while the other cooperates, the defector obtains the payoff T and the cooperator obtains the payoff S. The values of payoffs satisfy $T > R > S > P$. Thus in the SG, the best strategy depends on the opponent: to defect if the other cooperates, but to cooperate if the other defects. The combination of complex network and social dynamical behaviours has received much attention in the past few years and the snowdrift game based on the network has been widely studied.[8−19] Many different types of networks, i.e. scale-free Barabási–Albert networks,[20] Watts–Strogatz small-world network[21] and regular network have been used to study the evolutionary games.[22] A surprising finding is that the spatial structure often inhibits the evolution of cooperation in the snowdrift game.[8] While the studies on the scale-free networks show that increasing heterogeneity of the network is favourable for the emergence of cooperation.[10,11] It is observed that the cooperation is inhibited sometimes but enhanced in other cases, where the differences are due to different update rules and network contacts.[23]

In this paper, we study the evolutionary SG game in a heterogeneous Newman–Watts small-world network (HNW).[24] As is known, the Newman–Watts (NW) small-world network[25] is moderately homogeneous, with a Poisson degree distribution. In the NW network, some random 'shortcuts' are added to a regular ring graph. To introduce the heterogeneity into the NW model, we choose N_h vertices at random from all N nodes and treat them as hubs. Then we add m shortcuts by connecting one node randomly chosen from all N nodes to another node randomly chosen from N_h hubs (duplicate connections and self-links are voided). When $N_h = 1$, there is only one

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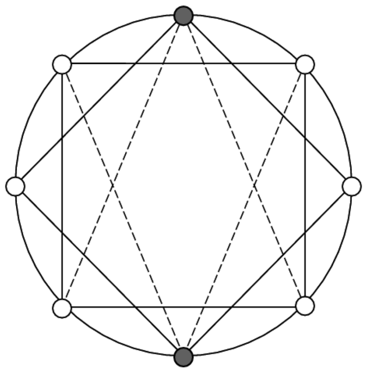
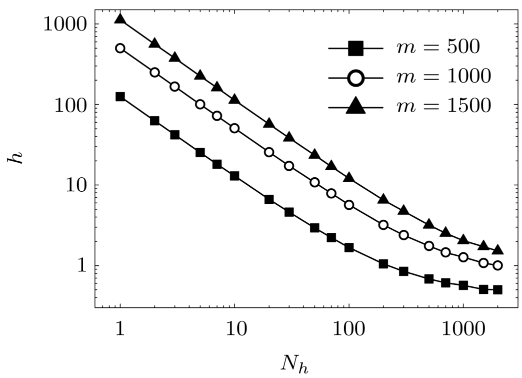
†E-mail: hxyang@mail.ustc.edu.cn

‡E-mail: bhwang@ustc.edu.cn

hub, and all shortcuts will connect to this node, which makes its degree very large. While $N_h = N$, shortcuts are added uniformly, which reduces the network to the original NW model. By changing the value of N_h, we can adjust the heterogeneity of the network. A sketch map of the HNW network is shown in Fig.1(a). The heterogeneity of the network is weighed by $h = N^{-1}\sum_k k^2 N(k) - \langle k\rangle^2$, where $N(k)$ is the number of nodes with k edges, $\langle k\rangle$ is the average degree of the network, and N is the network size.[24] Figure 1(b) shows that the degree of heterogeneity h decreases as the value of N_h increases.



(a)



(b)

Fig.1. (a)Illustration of the HNW network. Initially we construct a regular ring with each node connected to its nearest κ neighbours on each side (here $\kappa = 2$). The black dots are chosen as hubs. Then we add shortcuts. Each shortcut (dashed line) has at least one endpoint belonging to these hubs. (b) The degree of heterogeneity h vs N_h for different values of m. Network size $N = 2000$, each data point is averaged over 10 different realizations.

2. Simulation results

In the evolutionary SG game for the HNW network, each player occupies a node of the network, at each time step each player obtains his total payoff through playing the game with all his immediate neighbours. As proposed in Ref.[10], after each round, each individual, say x, randomly selects a neighbour y. When $P_y > P_x$, individual x will adopt the strategy of individual y with probability $(P_y - P_x)/Dk_>$, where P_x and P_y is the total payoff for x and y in the current time step respectively; $k_>$ is the larger one between the degrees of x and y; and $D = T - P$. In the evolution process, all individuals update their strategies synchronously. Following the previous approaches, we have $T = 1 + r$, $R = 1$, $S = 1 - r$ and $P = 0$, in which $0 < r < 1$. In all simulations below, the population of HNW network $N = 2000$. Each node on the regular ring has 2 neighbours on each side initially. The data each are obtained from averaging over 100 individual runs.

We firstly investigate the frequency of cooperation, f_c, as a function of r with different values of N_h. f_c is defined as the proportion of cooperators among all the players. Initially, cooperators and defectors are uniformly distributed among the population with equal possibility. f_c is obtained by averaging over the last 2000 time steps out of total 10000 time steps in evolution. In Fig.2(a), one can observe that the frequency of cooperation is not positively or negatively correlated with the N_h. Furthermore, in Figs.2(b) and 2(c) we find that there exists an optimal value of N_h corresponding to the highest level of cooperation for $r > 0.1$. When $r \leq 0.1$, f_c will reach 1 with the value of N_h increasing. As shown in Figs.2(b) and 2(c), for identical m, the value of the optimal N_h decreases as r increases. One the other hand, for identical r, the optimal value of N_h increases as m increases. We will explain the emergence of the optimal N_h in the following discussion.

Since the total payoff for a hub is usually much higher than those for other nodes, hubs can play an important role in the evolutionary process. It is worth our while to study the effect of hubs on the HNW network. We first consider the cases in which $N_h = 1$ and $N_h = 2$. It is shown in Fig.3 that under such extremely heterogeneous cases, the initial strategies of the hubs and the connection between hubs can almost determine the final state of the system. For $N_h = 1$, if the initial strategy of this hub is a cooperator (C-hub), the frequency of cooperation for the whole population can reach a higher value. On the other hand, if initially this hub is a defector (D-hub), f_c will go to a lower value. For $N_h = 2$, not only the initial strategies of the hubs, but also the correlation between these two hubs can influence the frequency of cooperation. If initially one hub is a cooperator and the other is a defector, the frequency of cooperation for the whole population extremely depends on whether they are connected with each other. If there exists a connection between these two hubs, they will finally be the same as the C-hubs. While if there is no connection, f_c will reach a lower value.

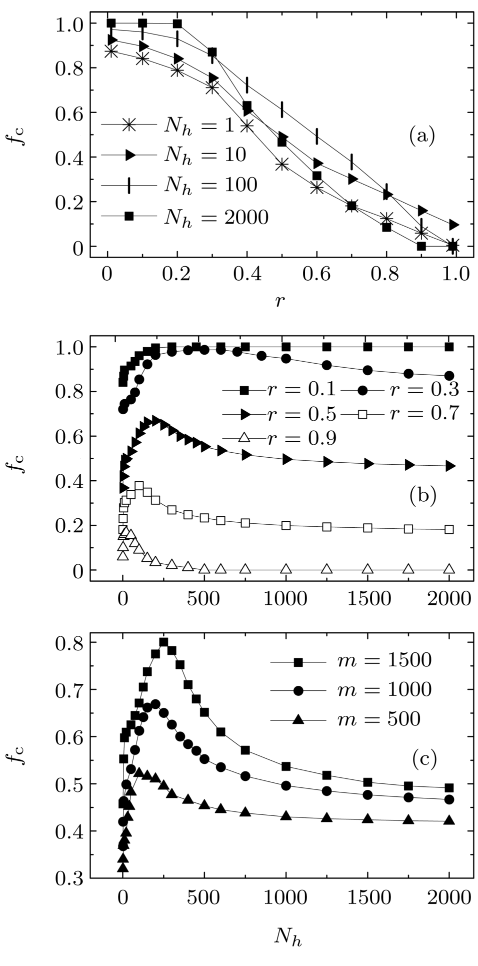


Fig.2. (a) The frequency of cooperation f_c vs r with different values of N_h and $m = 1000$; the frequency of cooperation f_c as a function of N_h with different values of r and $m = 1000$ (b), and different values of m and $r = 0.5$ (c).

In Fig.4, we further study how the connection between two hubs influences the evolution of f_c in the HNW network. Initially, one C-hub is connected to one D-hub. Since the hubs obtain higher payoffs, their neighbours are likely to learn their strategy. Therefore, it will form clusters around the hubs, in which players have the same strategy as their central hub. In Fig.4, one can see the value of f_c in Fig.4(b) is initially much lower than in Fig.4(a). It is easy to understand that the hub in a C-cluster can obtain a more payoff than the hub in D-cluster. The D-hub would change its strategy to cooperate since it is connected to the C-hub. After that, the whole D-cluster turns to a C-cluster. As a result, f_c in Fig.4(a) and f_c in Fig.4(b) reach the same levels.

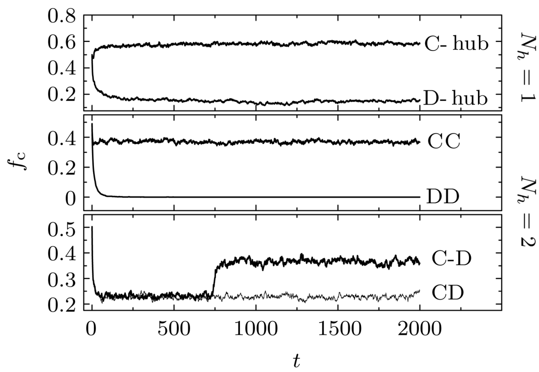


Fig.3. The evolutions of f_c with time. In (a) $N_h = 1, r = 0.5$ and $m = 1000$. C-hub means that this hub is a cooperator initially and D-hub means that this hub is a defector initially. In (b) and (c) $N_h = 2, r = 0.7$ and $m = 1000$. CC (DD) means that initially both hubs are cooperators (defectors); CD means that initially one hub is a cooperator, the other is a defector, and they are not connected; while C-D means they are connected.

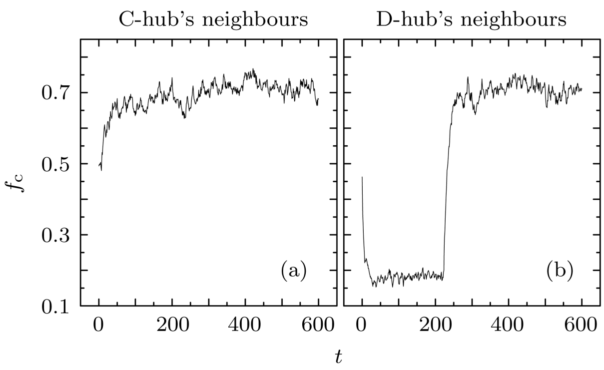
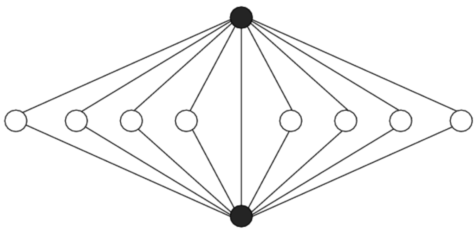
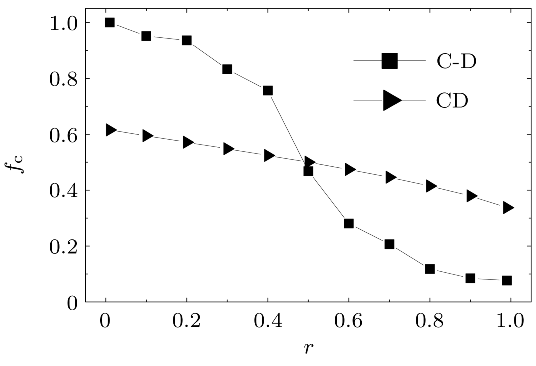


Fig.4. Evolutions of f_c of hubs' neighbours with time, in which $m = 1000, r = 0.7$, and $N_h = 2$, and a C-hub and a D-hub are connected with each other initially.

It should be pointed out that the connection between the C-hub and the D-hub does not always promote the cooperation in the snowdrift game. It depends on the network structure. As shown in Fig.5(a), a couple of connected C-hub and D-hub share the same n unconnected neighbours. The C-hub's payoff is $P_c = n[\rho_c * 1 + (1-\rho_c) * (1-r)]$, and the D-hub's payoff is $P_d = n\rho_c * (1+r)$, where ρ_c is the percentage of cooperators. When the degrees of the hubs are large enough, the probability for the neighbours to learn from these two hubs is approximately equal. As a result, $\rho_c \approx 0.5$. Then $P_c = n(2-r)/2$ and $P_d = n(1+r)/2$. If $2-r > 1+r$, where $r < 0.5$, then $P_c > P_d$, and the D-hub has the tendency to become a cooperator, which makes all the population convert to cooperators. On the contrary, if $r > 0.5$, then $P_c < P_d$, and the C-hub has the tendency to become a defector, which makes the rest of cooperators vanish in the network with two D-hubs. This conclusion can be confirmed in Fig.5(b). One can see that when $r < 0.5$, f_c for connecting hubs is higher than that for disconnecting hubs. While $r > 0.5$, situations are just on the contrary.



(a)



(b)

Fig.5. (a) Illustration of a subgraph, in which two connected hubs (black dots) have the same neighbours. (b) Frequency of cooperation vs r for the network shown in (a). Initially one hub is a cooperator, the other hub is a defector, and the two hubs share 1000 neighbours, whose initial strategies are randomly assigned. f_c is obtained by averaging over the last 1000 time steps of total 10000 time steps and each data point results from averaging over 100 simulations. C-D represents that two hubs are connected, and CD means that two hubs are disconnected.

Figure 6 shows that increasing the proportion of initial C-hubs and adding connections between hubs can promote the cooperation in the HNW network. On the contrary, decreasing C-hubs and connections between hubs will encourage the invasion of defectors. This finding is consistent with the previous result of the scale-free Barabási-Albert network.[10] It has been reported that interconnections between the hubs are favourable for the dominance of cooperation in the scale-free network.

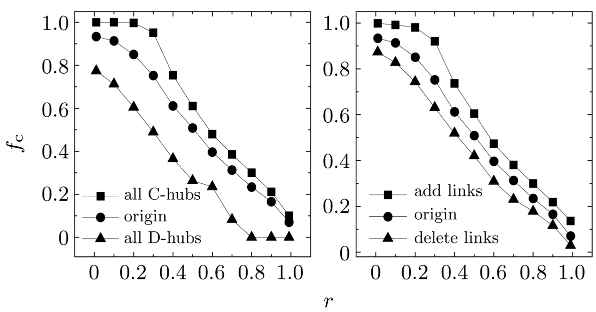


Fig.6. Frequency of cooperation vs r under different conditions. In the original graph, $N_h = 24$ and $m = 1000$. According to the origin graph, we change the initial strategies of hubs and the connections between hubs. Left panel shows that 'all C-hubs' represents that initially all hubs are cooperators and 'all D-hubs' represents that initially all hubs are defectors. Right panel shows that 'add links' represents that we add 25 edges between hubs, and 'delete links' represents that we delete all the edges between hubs. (we have checked that the number of links between 24 hubs is about 15, all our operations are ensured to make the graph connected).

From the above simulations and discussions, now we can know why there exists a moderate value of N_h corresponding to the highest level of cooperation. For large values of N_h, the network tends to be homogeneous and unfavourable for the cooperative behaviour. While for very small values of N_h, for example, $N_h = 1$, the network becomes extremely heterogeneous, which does not promote cooperation either. Since initially hubs can be a cooperator or a defector with equal possibility, we cannot ensure that initially all the hubs are cooperators in each run, and the emergence of D-hubs will make the frequency of cooperation reach a low value. For example, when $N_h = 1$, the probability of D-hub is about 0.5, as a result, the frequency of cooperation in half the runs will be very low. For a moderate value of N_h, though inevitably there are some D-hubs in the network at the beginning, many connections between hubs will make these D-hubs have more chances to learn from C-hubs, which promotes the spread of cooperators. For a fixed value of N_h, as the number of shortcuts increases, the degree of hub becomes larger, and the network turns more heterogeneous. The optimal value of N_h corresponding to $m = 1000$ makes the network too heterogeneous when $m = 1500$, so the optimal value of N_h for $m = 1500$ is larger than for $m = 1000$. To the contrary, the optimal value of N_h for $m = 500$ is smaller than for $m = 1000$. On the other hand, for a fixed value of m, as r increases, the payoff for the hub with the strategy of cooperation decreases, and the influence of C-hub is weakened. As a result, C-hubs cannot effectively prevent the defectors from invading. For the larger values of r, the C-hub needs more neighbours to ensure that its payoff is large enough to promote the cooperation. So the optimal value of N_h decreases with r increasing.

3. Conclusions

We have studied the snowdrift game in a heterogeneous Newman–Watts small-world network. It is interesting to find that the frequency of cooperation peaks at a middle value of N_h for a fixed value of r. That means that the moderate heterogeneity of the HNW network can promote the cooperation best. The previous studies have reported that increasing heterogeneity is favourable for the emergence of cooperation. While our studies of the HNW network show that the extremely heterogeneous case inhibits the cooperation in the snowdrift game. We also investigate how the initial strategies of hubs and the connections between hubs influence the evolution of cooperative behaviour.

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