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Evolutionary Prisoner's Dilemma Game Based on Pursuing Higher Average Payoff *

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We investigate the prisoner's dilemma game based on a new rule: players will change their current strategies to opposite strategies with some probability if their neighbours' average payoffs are higher than theirs. Compared with the cases on regular lattices (RL) and Newman–Watts small world network (NW), cooperation can be best enhanced on the scale-free Barabási–Albert network (BA). It is found that cooperators are dispersive on RL network, which is different from previously reported results that cooperators will form large clusters to resist the invasion of defectors. Cooperative behaviours on the BA network are discussed in detail. It is found that large-degree individuals have lower cooperation level and gain higher average payoffs than that of small-degree individuals. In addition, we find that small-degree individuals more frequently change strategies than do large-degree individuals.

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Cooperative behaviour is widely existent in many biological, social and economic systems.[1] Understanding the emergence and persistence of cooperation has attracted many natural and social scientists.[2] So far, game theory has provided a useful mathematical framework to characterize and to investigate the evolution of cooperation.[3−5] The prisoner's dilemma game (PDG)[6], as a famous model, is often used in this field. In the original PDG, two players simultaneously decide whether to cooperate (C) or defect (D) to gain some payoffs. They both receive R upon mutual cooperation and P upon mutual defection. If one defects while the other cooperates, the defector gets T and the cooperator gets S. The payoff matrix can be described as

	C	D
C	R	S
D	T	P

The values of four payoff satisfy $T > R > P > S$. Thus in the PDG, it is better for player to choose defection no matter what strategy his opponent adopts.

The spatial game, introduced by Nowak and May,[7] is a typical extension, which can result in emergence and persistence of cooperation in the PDG. Many interests have been given to spatial PDG in recent years.[8−30] discovered that scale-free networks provide a unified framework for the emergence of cooperation. Tang *et al.*[12] studied the effect of average degree on cooperation in networked evolutionary games. Ren *et al.*[13] studied the effects of both the topological randomness and the dynamical randomness on the evolutionary PDG and found that there exists an optimal amount of randomness, leading to the highest level of cooperation. Rong *et al.*[15] investigated how the degree-mixing pattern affects the emergence of cooperation. Fu *et al.*[17] found that there exists optimal cooperation level at intermediate topological heterogeneity.

It has been well accepted that the updating rule plays an important role in the evolution of cooperation. Tit-for-tat[7], win-stay-lose-shift (WSLS)[8] and the stochastic evolutionary rule introduced by Szabó *et al.*[11] are the commonly used rules in the evolutionary game. In addition, Wang *et al.*[31] proposed a self-questioning and memory-based mechanism to update strategies. Chen *et al.*[16] adopted a new rule in which players update their strategies according to aspiration payoff, and found that there exists an appropriate intermediate aspiration level leading to the maximum value of cooperation. Huang *et al.*[28] introduced birth-death and birth mechanisms to study the co-evolutionary dynamics of networks and games.

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In this Letter, we propose a new evolutionary game rule, in which players update their strategies based on pursuing higher average payoff than their neighbours'. After a round is over, each player gets total payoff through playing the game with all immediate neighbours. As used in Refs. [12,29,30], we evaluate the success (or fitness) of the players by their average payoff: total payoff divided by their connectivity degree. After each time step, player x will randomly select a neighbour y for possibly updating its strategy. Whenever $P_y > P_x$, player x will change its current strategy to its opposite strategy with probability given by

$$W = \frac{P_y - P_x}{T - S}, \qquad (2)$$

where P_x and P_y respectively represent the average payoff of player x and y. Following previous works, we set $T = b$, $R = 1$, $P = S = 0$, where $1 < b < 2$. We have checked that the simulation results do not change when making $S = -\varepsilon$ $(0 < \varepsilon \ll 1)$.

This evolutionary rule is stochastic in comparison with the deterministic switch in WSLS. Different from the stochastic evolutionary rule introduced by Szabó *et al.*, players do not share strategy information, they update strategies according to their own and their neighbours' payoff information.

One of important quantities for characterizing the cooperative behaviour is the density of cooperators ρ_c, which is defined as the proportion of cooperators among all the players. In following simulations, ρ_c is obtained by averaging from last 5000 Monte Carlo (MC) time steps of total 10000 MC time steps, where the system has reached a steady state. In the initial states, cooperators and defectors are uniformly distributed among all the players.

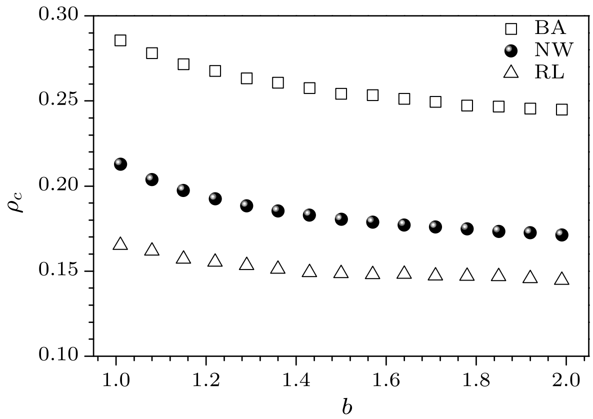

Fig. 1. Density of cooperators ρ_c as a function of b for different network structures with average connectivity $\langle k \rangle = 4$. The network size is $N = 10000$. Each data point is obtained by averaging over 20 different network realizations.

We apply the new updating rule to investigate the density of cooperators ρ_c on three typical network models: regular lattices (RL) with periodic boundary conditions, Newman–Watts small world network (NW)[32] and scale-free Barabási–Albert network (BA).[33] As shown in Fig. 1, BA network can best promote cooperation, while the cooperation level of RL network is the lowest. The differences depend on the spatial structure. As is well-known, RL network is the most homogeneous model with all nodes having the same degree, while the BA model is heterogeneous network with degree distribution accords with power law: $P(k) \sim k^{-\gamma}$, and the heterogeneity of the NW model is moderate. In previous studies, cooperators often go extinct for large b, while our rule can avoid cooperators vanishing even b is very large. From Fig. 1, one can observe that the frequency of cooperation f_c varies very slowly over the entire ranges of b on all the three networks.

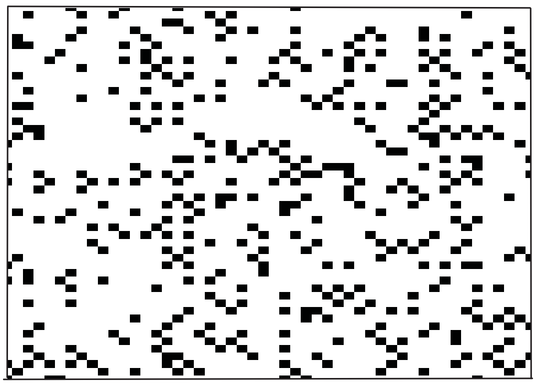

Fig. 2. A snapshot of 50×50 intercepted from the full 100×100 square lattice with four neighbours. Cooperator is in black and defector is in white. Time step $t = 8000$, $b = 1.5$.

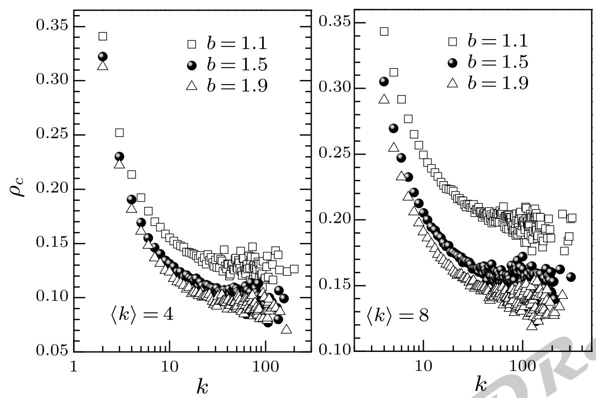

Fig. 3. Density of cooperators ρ_c, as a function of degree k on BA network. Network size $N = 10000$. Each data point is obtained by averaging over 20 different network realizations. Left panel: average degree $\langle k \rangle = 4$ with $b = 1.1$, 1.5, 1.9 respectively. Right panel: average degree $\langle k \rangle = 8$ with b=1.1, 1.5, 1.9, respectively.

In previous studies on the spatial PDG, cooperators can survive by forming large, compact clusters. While for the spatial PDG based on pursuing higher average payoff, cooperators are dispersive on the square lattice (see Fig. 2). Under the new updating rule, players do not adopt neighbours' strategies, instead they update strategies based on self-questioning mechanism: change current strategies to opposite strategies with some probability if their neighbours gain higher average payoffs. Since players do not change their strategies synchronously and there is no strategy intercommunion among them, it becomes very difficult to achieve strategy consensus in large-scale area.

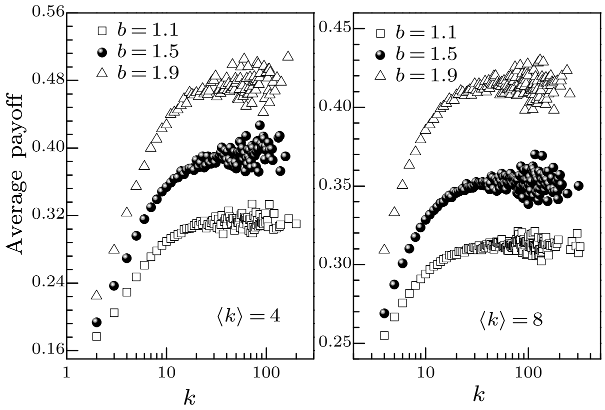

Fig. 4. Degree-dependent average payoffs of the players on BA network. Network size $N = 10000$. Each data point is obtained by averaging over 20 different network realizations. Left panel: average degree $\langle k \rangle = 4$ with $b = 1.1$, 1.5, 1.9, respectively. Right panel: average degree $\langle k \rangle = 8$ with $b = 1.1$, 1.5, 1.9, respectively.

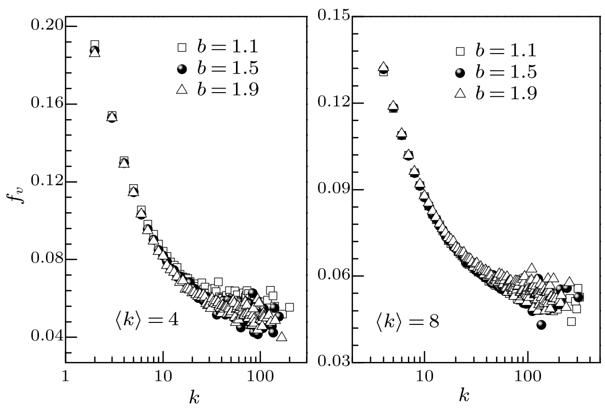

Fig. 5. Frequency of varying strategy f_v, as a function of degree k on BA network. Network size $N = 10000$. Each data point is obtained by averaging over 20 different network realizations. Left panel: average degree $\langle k \rangle = 4$ with $b = 1.1$, 1.5, 1.9, respectively. Right panel: average degree $\langle k \rangle = 8$ with $b = 1.1$, 1.5, 1.9, respectively.

In the following, we focus on cooperative behaviour on BA network. A series of simulation results show that connecting degree plays an important role in the evolution of cooperation.

Figure 3 shows that the density of cooperators ρ_c is negatively relative to connecting degree: large-degree individuals, i.e. the so-called hubs, have lower cooperation level than small-degree individuals. Figure 4 exhibits an opposite trend that large-degree individuals gain higher average payoffs than small-degree individuals. It is because defectors usually can gain higher payoff than cooperators in the PDG. To understand the effect of connecting degree on cooperative behaviour, we further investigate the frequency of varying strategy f_v, which is defined as the fraction of varying strategy times in total time steps. From Fig. 5, one can find that f_v is almost independent of b and small-degree individuals are more frequent to change their strategies than large-degree individuals. Lacking enough neighbours makes small-degree individuals easily influenced by surrounding environment, any shift of neighbours' strategies will impact their average payoff remarkably. Contrarily, the average payoffs of large-degree individuals are steady since they have large number of neighbours and these neighbours do not change strategies synchronously. Since defection usually bring more payoff than cooperation in PDG, players tend to take defection. There always exist some C-neighbours around the D-hubs, so the D-hubs can gain payoffs and keep defection for a long time. On the other hand, many small-degree nodes are connected to the hubs. No matter what strategy small-degree individuals adopt, their average payoffs are often very low because most of their neighbours are defectors. Small-degree individuals have to change strategies more frequently since they often have lower average payoffs than their D-hub neighbours. As a result, small-degree individuals have higher cooperation level.

In conclusion, we have studied the prisoner's dilemma game (PDG) based on a new updating rule, in which players change their strategies to opposite strategies with some probability if they find that their neighbours' average payoffs are higher than theirs. The new rule can protect cooperators from extinction even b is large. It is found that cooperators are dispersive on square lattice. We further investigate cooperative behaviour on the BA network and find that connecting degree plays an important role in networked games. Compared with small-degree individuals, large-degree individuals have lower cooperation level and gain higher average payoffs. Besides, we find that small-degree individuals change their strategies more frequently. Our work may be helpful in exploring the role of updating rule in the evolutionary game, and can contribute to study the cooperative behaviour in realistic world.

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