Evolutionary Prisoner's Dilemma Game Based on Division of Work *

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We propose a new two-type-player prisoner's dilemma game based on division of work on a square lattice, in which a fraction of the population μ are assigned type A and the rest B. In a one-shot two-player game, we let both of their original payoffs scaled by a same multiplicative factor $\alpha > 1$, if two neighboring players are of different types; while leave the payoffs unchanged, if they are of the same type. Then we show that combined with the two-type setup, the square lattice can assist to induce different social ranks according to players' abilities to collect payoffs. Simulation results show that the density of cooperation is significantly promoted for a wide range of the temptation to defection parameter and that there are optimal values for both α and μ leading to the maximal cooperation level. We understand these results by analyzing the distribution of the players in the social ranks and we also show some typical snapshots of the system.

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Evolutionary game theory has been considered an important approach to characterize and understand the emergence of cooperative behavior in systems consisting of selfish individuals.[1,2] Such systems are ubiquitous in nature, ranging from biological to economic and social systems. The evolutionary prisoner's dilemma game (PDG) as a general metaphor for studying the cooperative behavior has drawn much attention from scientific communities.[3] In the original PDG, two players simultaneously decide whether to cooperate or defect. They will receive R if both cooperate, and P if both defect. While a player receives S when confronted to a defector, which in turn receives T, where $T > R > P > S$. Evidently, for one-shot PDG defection is unbeatable and thus preferred by rational players, although they can realize that mutual cooperation yields higher payoff than mutual defection. This inhibited cooperative behavior is contrary to the empirical evidence and motivates a number of extensions of the original games to provide better explanations for the emergence of cooperation.

The spatial game, introduced by Nowak and May,[4] is a typical extension, in which cooperators are able to survive via clustering that protects them mutually against exploitation by invading defectors. In recent years, topological inhomogeneities have been introduced to promote the level of cooperation,[5−17] whereby in particular the scale-free network has been identified as an excellent host for cooperative individuals. In the scale-free network, because of strong inhomogeneity of the degree of the nodes, a few players located on high degree nodes (called hubs) collects much higher payoffs and becomes an example to be followed by their neighbors. Besides the topological inhomogeneities, inhomogeneities of individual personality have also been introduced. Among such approaches, social diversity has been shown to be able to promote cooperation.[19−21] Another example considers two types of players with asymmetric teaching and learning activities,[22−27] in which some distinguished players with stronger capacity of spreading their own strategies lead to the thrive of cooperation.

When we ponder the roots of cooperation, we should note that in real economic life, one prerequisite for cooperation is division of work, because production efficiency increases greatly and cooperation becomes meaningful provided that the production process is decomposed into several different parts and fulfilled by specialized skillful workers. The phenomenon of division of work has not been considered in previous work. In this Letter, motivated by the above facts, we modify the classical spatial game[18] by dividing the population into different work types which coincide with the types of players, so that interplays between different work types (player types) acquire higher productions (payoffs). For the sake of simplicity, we constraint there to be only two different work types. Note that in a one-shot game if the payoffs are enhanced, the two different types equally benefit. However, there is an inhomogeneity among different one-shot games: The payoffs of some one-shot games are enhanced by the parameter $\alpha > 1$, while the others are not, depending on the type combinations of the two players. We show in the following that the spatial lattice plays

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an decisive role in cumulating the inhomogeneity in one-shot games and enlarging the payoff differences among players, which greatly affects the cooperative behavior of the system.

In this model, before the simulation two types of players (A and B) are located on the sites of a square lattice with a concentration of μ and $(1-\mu)$, respectively. Their random initial distribution is fixed during the simulations. Both types of players following the pure strategies: either C or D, play the PDG with their nearest neighbors and collect the total payoffs according to the parameterized payoff matrix suggested by Nowak and May.[4] The setup of two-type-player effects as follows: When two neighboring players play the game, if they are of the same type (namely, both A or both B), the payoff matrix is set to be $T = b > 1$, $R = 1$, and $S = P = 0$; while if their types differ, the matrix is scaled by a multiplicative factor $\alpha > 1$,[19] becoming $T = \alpha \cdot b$, $R = \alpha$, and $S = P = 0$. Evidently this scaling preserves the payoff ranking of the PDG. This setting of the payoff matrices can be interpreted as that the production efficiency actually equals 1 for interactions between two same work-types, while the production efficiency is enhanced to be α for two different types.

After each full cycle of the game, the performance of player x is compared with that of a randomly chosen neighbor y and the probability that its strategy changes to s_y is given by

$$W(s_y \to s_x) = \frac{1}{1 + \exp\left[(P_x - P_y)/K\right]},$$

where $K = 0.1$ characterizes the uncertainty related to the strategy adoption process. Note that when $\mu = 0$ or 1, then only one type of players left and the model returns to the classical spatial game. When α approaches 1, the introduction of division of work cease to effect and the model also returns to the classical one. Besides, this two-type model has a symmetry that when we switch each player's type to the other type (meanwhile the value of μ is changed into $(1-\mu)$), the evolutionary result should be invariant. Thus we only need to consider about the interval of $\mu \in [0, 0.5]$, subsequently.

Before showing the simulation results, we would like to analyze the varied status of the players in the population presently. For simplicity, we call two neighboring players to be complementary if their types are different, by which we define the social rank of a player as the number of its complementary neighbors. When a player plays games with its neighbors, the higher its rank, the more payoffs obtained in a one-shot game are enhanced by the parameter α, and then the stronger ability to collect payoffs. Thus the rank of one player is understood as its ability to collect payoffs. Since the degree of the nodes in the square lattice all equal 4, obviously, one player may have either 0, 1, 2, 3 or 4 complementary neighbors, which determines the player's rank accordingly. Thus, starting from two types of players, we now reach a point that there exists five unequal social ranks. We note that the gaps among each ranks depend linearly on α and the definition of social rank makes sense especially when α is much larger than 1.

The simulations are performed on a square lattice with periodic boundary conditions and of size $N = 100 \times 100$. The evolution of the spatial distribution of the C and D strategies starts from an uncorrelated initial state where cooperators and defectors are present with the same probability. The key quantity for characterizing the cooperative behavior of the system is the density of cooperators ρ_c, which is defined as the fraction of cooperators in the whole population. In all simulations, ρ_c is obtained by averaging over the last 2000 generations of the entire 10000 generations. Each data point results from an average of 20 runs from each set of independent initial parameter values and the above model is simulated with synchronous update.

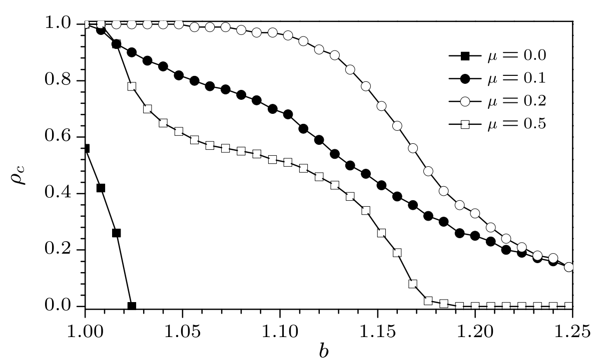

Fig. 1. Density of cooperators ρ_c as a function of temptation to defect b for different values of μ when $\alpha = 3.0$.

First, we present ρ_c in dependence on b for different values of μ, as shown in Fig. 1. For $\mu = 0$, the model returns homogeneous and the cooperation level decays fastest. However, in contrast, for other values of μ when the system is inhomogeneous, the cooperation level is profoundly enhanced for a wide range of values of b. In order to examine the impact of the parameter μ, we study ρ_c in dependence on μ for different values of α by setting $b = 1.03$, as shown in Fig. 2. At a glance, these curves are symmetric about the middle line, which confirms our previous assertion about the symmetry in the model. Within the range of $0 \leq \mu \leq 0.5$, ρ_c increases monotonically with μ until reaching its maximum value, then ρ_c decreases with μ until the middle point at $\mu = 0.5$. Similar non-monotonic dependence of ρ_c on the fraction of the type A players μ was also found in the previous two-type-player model.[24,25] One can also find that different values of α can influence the cooperation level for fixed μ, so there may exist some appropriate value for α promoting cooperation. Besides, the optimal value of μ guaranteeing the best cooperation level is also affected by α: For smaller values of $\alpha = 1.5$ and 2.0, the optimal value is at $\mu \simeq 0.2$; while for larger values of $\alpha = 3.5$ and 4.5, the optimal value is shifted to $\mu \simeq 0.3$.

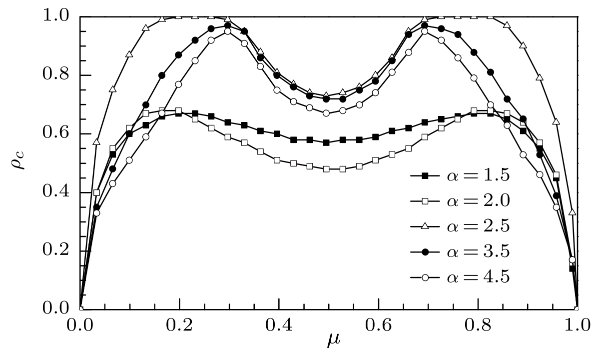

Fig. 2. Density of cooperators ρ_c in dependence on the fraction of type A players μ for different values of α when $b = 1.03$.

As to the reasons for the cooperative trait shown above, we argue that the types of the players A and B are actually dummy, rather it is the induced diversified social ranks that plays the essential role in the facilitation of cooperation. To confirm this argument, we refer to the previous work by Perc *et al.*[19] In that work, the authors introduced the diversity in players' payoffs into the classical spatial PDG and showed that the diversity can promote cooperation. They also accounted for that observation by the so called feedback mechanism. Specifically, the high-ranking players (i.e. players' with high payoffs) can dominate their neighbors, eventually forming large clusters around them with a given strategy. During this process the lower-ranking players can form an obedient domain around a high-ranking player and so prevail against defectors, while a high-ranking defector will be weakened by the low-ranking players who follow its destructive strategy. Thus eventually the victory of a high-ranking cooperator against a high-ranking defector is more favorable. While in the present model, we showed previously that there exists exactly 5 social ranks, so similar clustering and feedback mechanisms also take effect, which lead to the significant promotion of cooperation.

It is instructive to discuss in detail how the distributions of the social ranks affect the cooperative behavior.[19] In fact, by calculating the chances of different type-combinations of the players located on the square lattice, it is easy to give a so-called rank distribution formula for varied μ as follows:

$$n_i(\mu) = [(1-\mu)^i \mu^{5-i} + \mu^i (1-\mu)^{5-i}] N \binom{i}{4},$$

where $i = 0, 1, 2, 3, 4$ denotes the five ranks in the population, n_i denotes the number of players belong to rank i, $N = 10000$ is the size of the population and $\binom{i}{4}$ is the combination number. To have some intuitive feelings, we plot the histogram of the rank distribution for several typical values of the continuous variable μ in Fig. 3. By virtue of these histograms, firstly, one can find that the curves with significantly different trends for each value of μ in Fig. 1 are actually corresponding to the different distributions for the same value of μ in Fig. 3. Secondly, we may revisit the result obtained in Fig. 2. When $\mu = 0$, as shown in Fig. 3(a), all players occupy the lowest rank. This homogeneous social mode corresponds to the vanishing of ρ_c in Fig. 2. When $\mu = 0.1$, as shown in Fig. 3(b), most players occupy the two lowest ranks, while notably with a few players emerging in the three higher ranks. The emergence of the inhomogeneity in the population triggers the feedback mechanism and leads to a remarkable increase of ρ_c, as shown in Fig. 2. When $\mu = 0.2$ and 0.3, as shown in Fig. 3(c), more players fill the higher ranks. We can infer that, with the concentration of the high-ranking players increasing, the feedback mechanism is reinforced and this leads to a further increasing of ρ_c. However, when $\mu = 0.4$ and 0.5, as shown in Fig. 3(d), the rank distribution changes dramatically and most players occupy the three middle ranks. Meanwhile, a drop of cooperation level is observed in Fig. 2. These results indicate that the specific rank distribution has a significant impact on the evolution of the system.

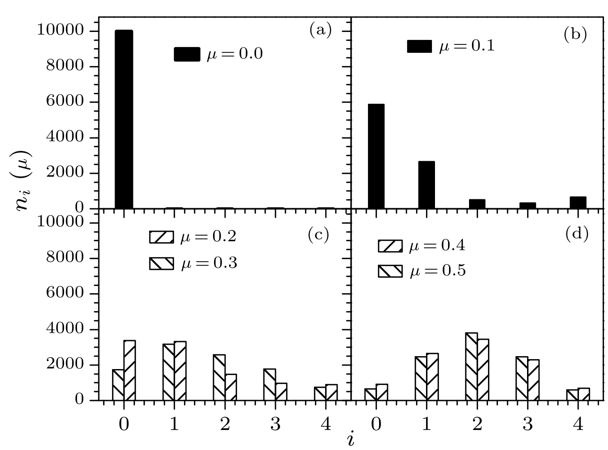

Fig. 3. Number of players n belong to rank i for different values of μ, classified into four groups on intuitive and each corresponding to a certain social mode: (a) homogeneous, (b) "large in bottom", (c) "pyramid styled" and (d) "large in the middle".

In the above discussion, we use μ to govern the distribution of the social ranks at the population level, while we have another parameter α to adjust the payoff gaps among each rank at the individual level. Thus we now focus on how ρ_c changes with α systematically. Results obtained for different b and by setting $\mu = 0.1$ are revealed in Fig. 4. Interestingly, one can see that ρ_c varies non-monotonically with α. There exists roughly two maximum values for each curve. This result indicates that moderately enlarging the gaps among the social ranks can facilitate cooperation, while exceedingly large gaps inhibit cooperation. The curve obtained at $\mu = 0.1$ is different from the other two. This is reasonable since the rank distributions for different values of μ do differ. We mention that, when α is exceedingly large, the payoffs obtained from interactions between two neighboring players with the same type can be neglected.

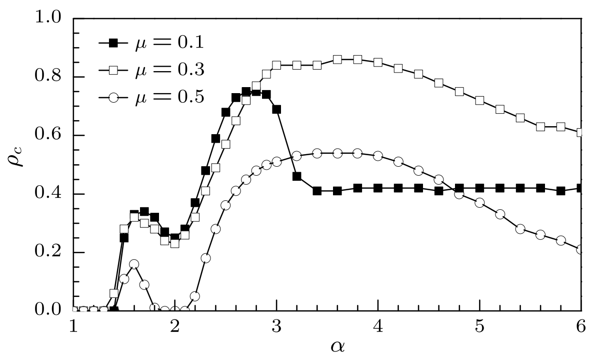

Fig. 4. Density of cooperators ρ_c vs α for different values of μ when $b = 1.1$.

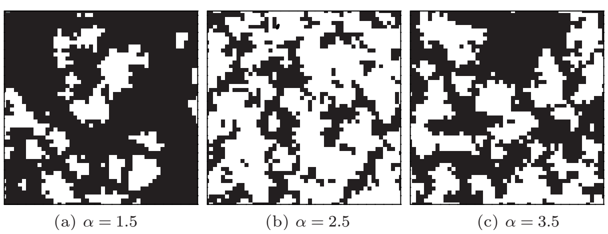

Fig. 5. Snapshots of typical distributions of cooperators (white) and defectors (black) on a square lattice when $b = 1.10$, $\mu = 0.1$ and for different values of α. These snapshots are a 50×50 portion of the full 100×100 lattice.

In order to intuitively understand the effect of the parameter α on cooperation, we plot some typical snapshots of the system at equilibrium for fixed $b = 1.10$ and $\mu = 0.1$ with respect to different α values, as shown in Fig. 5. One can find that, when α is slightly larger than 1, e.g., $\alpha = 1.5$ as shown in Fig. 5(a), cooperators form some small and isolated patches to resist the invasion of defectors. When α is larger, e.g., $\alpha = 2.5$ as shown in Fig. 5(b), there occurs more and larger cooperative clusters. It is indicated that enlarging the gaps among the social ranks can make the spreading of the strategies from higher-ranking players to the lower-ranking players more effective and thus reinforce the clustering and feedback mechanisms. Thus this leads to an increasing stage of ρ_c with α in Fig. 4. However, when α becomes even larger, e.g., $\alpha = 3.5$, as shown in Fig. 5(c), the size of the cooperative clusters decreases. Since cooperators are more likely to benefit from forming clusters than defectors, when α is further increased to exceed the optimal region, a drop of cooperation level is observed in Fig. 4.

In summary, we have proposed a new two-type-player PDG model on a square lattice based on division of work. We find that the spatial lattice together with the two-type setup can induce five social ranks satisfying certain rank distributions. Due to this inhomogeneity in the population, it is shown that the cooperation level is substantially promoted for a wide range of temptation to defection values. We find that maximal cooperation level is obtained when the proportion between the two types is quite imbalanced. Finally we find that the cooperation density relies on the factor α non-monotonically, which indicates that moderate gaps among the social ranks more favors cooperation.

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