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Preferential selection promotes cooperation in a spatial public goods game

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ABSTRACT

We introduce a preferential selection mechanism into a spatial public goods game where players are located on a square lattice. Each individual chooses one of its neighbors as a reference with a probability proportional to $\exp(P_y * A)$, where P_y is the neighbor's payoff and A (≥ 0) is a tunable parameter. It is shown that the introduction of such a preferential selection can remarkably promote the emergence of cooperation over a wide range of the multiplication factor. We find that the mean payoffs of cooperators along the boundary are higher than that of defectors and cooperators form larger clusters as A increases. The extinction thresholds of cooperators and defectors for different values of noise are also investigated.

1. Introduction

Cooperation plays an important role in the real world, ranging from biological systems to economic and as well as social systems [1]. Scientists from different fields of natural and social sciences often resort to evolutionary game theory as a common mathematical framework [2,3]. The prisoner's dilemma game (PDG) as a paradigm to explain cooperative behavior through pairwise interactions [4] has been widely studied [5–24]. As an alternative, the public goods game (PGG) for group interactions has also attracted much attention for studying the emergence of cooperative behavior [25]. In a typical PGG played by N individuals, each individual can choose to cooperate or defect. Cooperators contribute an amount c to the PGG, while defectors do not contribute. The total contribution is multiplied by a factor r, and is then redistributed uniformly among all players. It was shown that for the values of $r < N$, the defectors will dominate the whole population [26].

Many works have been done on the public goods game [27–35]. Szabó et al. have studied the voluntary participation in the PGG on a square lattice and found that the introduction of loners leads to a cyclic dominance of the strategies and promotes substantial levels of cooperation [28]. Guan et al. have confirmed that the introduction of the inhomogeneous activity of teaching the players can remarkably promote cooperation [32]. Very recently, Santos et al. have introduced social

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diversity by means of heterogeneous graphs, and it was shown that cooperation is promoted by the diversity associated with the number and size of the public goods game in which each individual participates and with the individual contribution to each such game [34].

In the present paper, we introduce a preferential selection into the spatial public goods game. The individuals do not completely randomly choose a neighbor to refer to. Instead, the probability of selecting one of its neighbors as a reference is non-uniform and depends on the neighbor's payoff. We have found that this preferential selection can promote the emergence of cooperation on a square lattice with periodic boundary conditions.

2. The model

We consider the evolutionary PGG on a square lattice with periodic boundary conditions. Players are only cooperators (C) or defectors (D). Each player interacts only with four nearest neighbors (von Neumann neighborhood), thus the size of interaction neighborhood is $g = 5$ (including the player and its four nearest neighbors). The player x's payoff is

$$P_x = \frac{rn_c}{g} - 1, \quad \text{if } s_x = C,$$
$$P_x = \frac{rn_c}{g} - 0, \quad \text{if } s_x = D, \tag{1}$$

where s_x denotes the strategy of the player x, r represents the multiplication factor and n_c is the number of cooperators in the neighborhood. Here the contribution of each cooperator is normalized to unity.

After each time step, each player is allowed to select one of its neighbors as a reference. Player x will select a neighbor y with a probability

$$Q_{x \to y} = \frac{\exp(P_y * A)}{\Sigma_{z \in \Omega_x} \exp(P_z * A)}, \tag{2}$$

where Ω_x is the set of x's neighbors and A is a tunable parameter. For $A > 0$ ($A < 0$), the neighbor with higher (lower) payoff has more probability to be selected. When $A = 0$, the neighbor is randomly selected as in previous works. Since, in a realistic world, the influence of people with low payoff is usually weak, in this paper, we only study the case of $A \geq 0$.

Player x will adopt the selected neighbor y's strategy in the next time step with a probability depending on their payoff difference, presented in Refs. [36–41], as

$$W(s_x \to s_y) = \frac{1}{1 + \exp[(P_x - P_y)/\kappa]}, \tag{3}$$

where κ denotes the amplitude of noise.

3. Simulation and analysis

Simulations were carried out for a population of $N = 100 \times 100$ individuals. Initially, the two strategies of C and D are randomly distributed among the individuals with equal probability, 1/2. The above model was simulated with synchronous updating. We have checked that no qualitative changes occur if an asynchronous updating is adopted. Eventually, the system reaches dynamic equilibrium state. We study the key quantity of cooperator density ρ_c in the steady state. In all our simulations ρ_c is obtained by averaging the last 2000 Monte Carlo (MC) time steps of the total 22 000. Each data point results from an average of over 30 realizations.

Fig. 1 shows that the cooperator density ρ_c as a function of the multiplication factor r for different values of A. One can see that ρ_c monotonically increases as r increases, no matter what the value of A is. This result is consistent with previous findings that larger r can better promote the emergence of cooperation. Compared with the random selection case, the introduction of preferential selection can remarkably promote the cooperative behavior over a wide range of r. For a fixed value of r, the cooperator density ρ_c increases as the value of A increases.

Notice that cooperators tend to form cluster patterns where cooperators assist each other in avoiding defectors' exploitation in spatial games [42,43]. A cooperator (defector) cluster is a connected component (subgraph) fully occupied by cooperators (defectors). In order to visualize the effect of A on cooperation, we plot some typical snapshots of the system at equilibrium for fixed $r = 4$ with respect to different values of A. Fig. 2 shows that the cooperator clusters become larger while the defector clusters become smaller as A increases.

Generally, the cluster formation influences the interactions between cooperators and defectors, and thus affects the payoffs of cooperators and defectors along the boundary. A cooperator (defector) is on the boundary if it has at least one defector (cooperator) neighbor. Fig. 3 shows the mean payoffs of cooperators and defectors along the boundary ($\overline{P}_{c\_bound}$ and $\overline{P}_{d\_bound}$) as a function of time step t. One can find that $\overline{P}_{c\_bound}$ always exceeds $\overline{P}_{d\_bound}$ during the whole evolutionary process. Fig. 4 shows that, in the equilibrium state, $\overline{P}_{c\_bound}$ is no less than $\overline{P}_{d\_bound}$ for various values of A. The mean payoffs of both cooperators and defectors along the boundary, $\overline{P}_{c\_bound}$ and $\overline{P}_{d\_bound}$, are zero when $A = 0$, since cooperators become

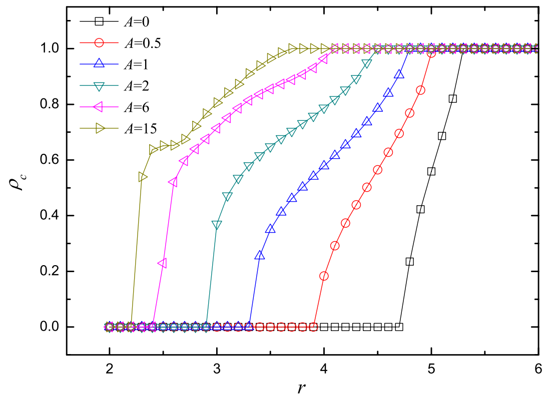

Fig. 1. (Color online). The cooperator density ρ_c as a function of the multiplication factor r for different values of A. The amplitude of noise $\kappa = 0.1$.

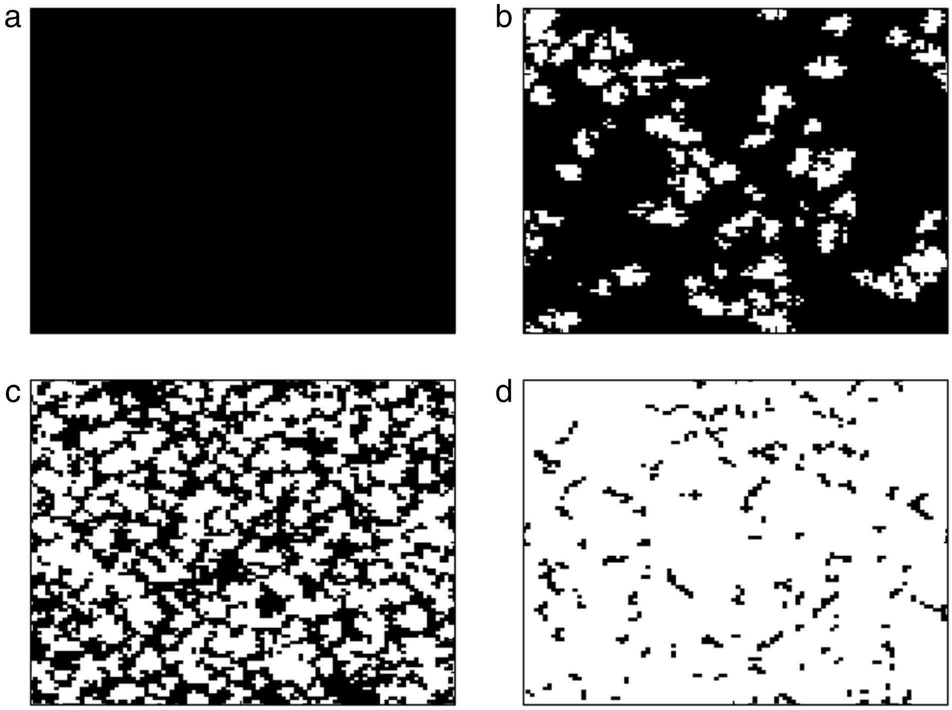

Fig. 2. A series of snapshots of distribution of cooperators (white) and detectors (black) on a 100×100 square lattice at $r = 4$, $\kappa = 0.1$ for four values of A: (a) $A = 0$, (b) $A = 0.5$, (c) $A = 1$ and (d) $A = 4$.

extinct in the population and there is no boundary between cooperators and defectors in this situation. The difference between $\overline{P}_{c\_bound}$ and $\overline{P}_{d\_bound}$ becomes smaller as A increases. For large values of A, defectors will form small filament-like clusters with a typical width of one lattice unit (see Fig. 2(d)); thus defectors along the boundary can touch more cooperators and $\overline{P}_{d\_bound}$ increases more quickly than $\overline{P}_{c\_bound}$.

Now the reason why the preferential selection mechanism promotes cooperation can be understood: Cooperators along the boundary usually have higher payoffs than those of defectors. As the value of A increases, cooperators along the boundary are more frequently selected as a reference, which favors the spreading of cooperators.

From Fig. 1, one can observe that below some threshold values of the multiplication factor ($r < r_c$), cooperators will vanish ($\rho_c = 0$), whereas for $r > r_d$, defectors go extinct ($\rho_c = 1$). Fig. 5 shows the threshold values of r_c and r_d as a function of A for different values of noise κ. It is shown that both r_c and r_d decrease as A increases for various values of κ.

4. Conclusion

In summary, we have studied the evolutionary public goods game on a square lattice. A preferential selection is introduced with the selection probability proportional to $\exp(P_y * A)$, where P_y is the neighbor's payoff and A (≥ 0) is a

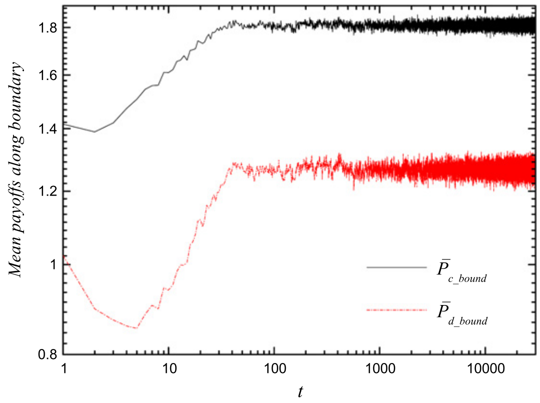

Fig. 3. (Color online). The mean payoffs of cooperators and defectors along the boundary ($\bar{P}_{c-bound}$ and $\bar{P}_{d-bound}$) as a function of time step t for $A = 1$, $r = 4.0$, $\kappa = 0.1$.

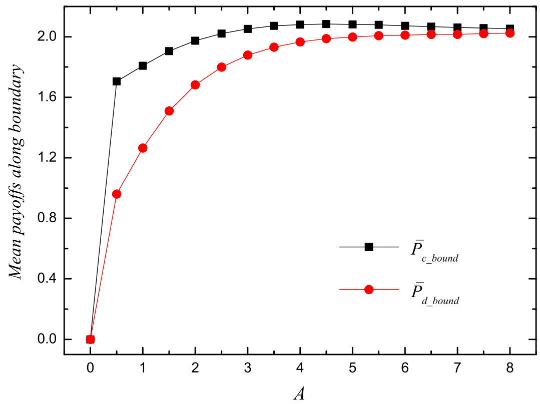

Fig. 4. (Color online). The mean payoffs of cooperators and defectors along the boundary ($\bar{P}_{c-bound}$ and $\bar{P}_{d-bound}$) in the equilibrium state, as a function of A for $r = 4$, $\kappa = 0.1$.

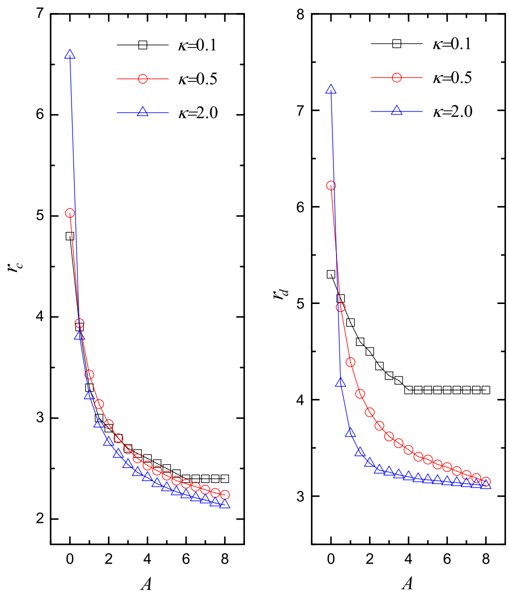

Fig. 5. (Color online). The cooperator (defector) extinction threshold $r_c(r_d)$ as a function of A for different values of noise κ.

tunable parameter. It is found that the cooperator density ρ_c increases as A increases. In order to give an intuitive account of the maintenance of cooperation, we provide some typical snapshots of the system and compare mean payoffs of defectors and cooperators along the boundary. It is shown that cooperators can survive by forming compact clusters, and along the boundary, cooperators can gain more payoffs than defectors can. Besides, we find that the extinction thresholds of both cooperators and defectors decrease as A increases for various values of the noise κ. That is to say, an increment of A increases the region of parameters r where cooperators survive or conquer the whole system.

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