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Randomness Effect on Cooperation in Memory-Based Snowdrift Game *

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A memory-based snowdrift game (MBSG) on spatial small-world networks is investigated. It is found that cooperation rate versus temptation shows some step structures on small-world networks, similar to the case on regular lattices. With the increment of rewiring probability based on four-neighbour regular lattices, more steps are observable. Interestingly, it is observed that cooperation rate peaks at a specific value of temptation, which indicates that properly encouraging selfish actions may lead to better cooperative behaviours in the MBSG on small-world networks. Memory effects are also discussed for different rewiring probabilities. Furthermore, optimal regions are found in the parameter planes. The strategy-related average degrees of individuals are helpful to understand the obtained results.

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Although unselfish, altruistic actions apparently contradict Darwinian selections, cooperation phenomena are abundant in nature, ranging from microbial interactions[1] to human behaviour.[2] Thus understanding the conditions and mechanisms for the emergence and maintenance of cooperative behaviour among selfish individuals becomes a central issue.[2,3] Evolutionary game theory has become an important tool for investigating and understanding cooperative or altruistic behaviour in systems consisting of competitive entities, including biological, economic and social systems.[4] Since the well-known computer tournaments of Axelrod and their applications to biological communities,[5] much attention has been paid to the Prisoner's Dilemma Game (PDG) as a metaphor for the problems surrounding the evolution of cooperative behaviour. However, the PDG has some restrictions in discussing the emergence of cooperation, since it is better to choose defection as one's strategy no matter what strategy its opponent adopts. As a result, the proposal of the Snowdrift Game was introduced to be an alternative to the PDG.[6] In contrast to PDG, the snowdrift game favours cooperation because it is better for one to defect if the other cooperates, but to cooperate if the other defects. There still exists unstable cooperative behaviour which is contrary to the empirical evidence. This disagreement stimulates a number of extensions of the original games to provide better explanations for the emergence of cooperation.

Recently, Wang *et al.* introduced memory-based snowdrift game (MBSG) on both two-dimensional lattices and scale-free network,[7] on which players make decisions according to the specific probability which can be calculated by the information stored in their memories, based on the fact that individuals usually make decisions according to the knowledge of past records in nature and society, and that the historical memory would play a key role in an evolutionary game. It is found that memory effects play different roles on the frequency of cooperation for distinct ranges of the payoff parameter and that the frequency of cooperation decreases suddenly at a few specific values. In contrast to the previously reported results,[8] in which Hauert *et al.* presented that spatial structure inhibits the evolution of cooperation in the snowdrift game, Wang *et al.* showed that cooperation is promoted by the spatial structure in MBSG. More recently, Ren studied the evolutionary PDG on homogeneous small-world network.[9] It is found that there exists an optimal topological randomness p, which promotes cooperation at various values of the temptation to defect b, so that a resonance-type phenomenon appears.

Most of the real networks are characterized by the same topological properties, such as small average path lengths and high clustering coefficients.[10] Networks with both properties are defined as small-world networks by Watts and Strogatz.[11] In order to investigate how the MBSG evolve and how to promote cooperation in real-world networks, we focus on the MBSG on small-world networks to find what happens when the topological randomness p changes via nu-

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merical simulation. We found some nonmonotonous phenomena which do not occur in regular lattices. In comparison of results obtained from 4-neighbour regular lattices and small-world networks based on 4-neighbour regular lattices (SWNBRL), one can find that randomly rewiring of small-world networks has nontrivial effects on the evolutionary MBSG.

First, we briefly describe the original snowdrift game model. Imagine that two drivers are trapped on either side of a snowdrift. They can either get out of the car to shovel (cooperate C) or remain in the car (defect D). If both the cooperates, they can both gain benefit b of getting back home while sharing labor c. Thus both get payoff of $R = b - c/2$. If mutual defection, they will still be blocked and get payoff of $P = 0$. If only one shovels (C) while the other remains in car (D), then they both return home but the defector avoids the labor cost and gets a best payoff of $T = b$, whereas the cooperator takes on all the labor cost and gets a "sucker" payoff of $S = b - c$. Without loss of generality, R is usually set to be 1 so that the evolutionary behaviour of the snowdrift game can be investigated with a single parameter $r = c/2 = c/(2b - c)$ for the cost-to-reward ratio. Thus, one has a rescaled payoff matrix

$$\begin{array}{c} \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cc} C & D \end{array} \\ \begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix} \end{array},$$

where $0 < r < 1$. The best action depends on the opponent: to defect if the other cooperates, but to cooperate if the other defects.

Next, we present the rules of the evolutionary MBSG.[7] Suppose that N players placed on the nodes of a certain network structure play the game simultaneously in pairs in every round. The total payoff of each player is the sum over all its encounters. Once a round is over, each player will have the strategy information (C or D) of its encounters. Then, each player adopts its antistrategy to play a virtual game with all its opponents, and calculate the virtual total payoff. By comparing the virtual payoff with the actual payoff, each player can know its best strategy corresponding to the highest payoff and then record it into its memory. Assume that players can only remember the last M bits of the past strategy information. At the next generation, each player chooses C (D) as its strategy according to the probability $P_C = \dfrac{N_C}{N_C + N_D} = \dfrac{N_C}{M}$ ($P_D = 1 - P_C$), which depends on the ratio of the numbers of C (D) stored in its memory, where N_C and N_D are the numbers of C and D, respectively. Subsequently, all players update their memories simultaneously. Repeat the above process and hence the system evolves.

The first small-world network is proposed by Watts and Strogatz by introducing a randomly rewiring process, based on a regular ring graph.[11] Here, we adopt a modified Watts and Strogatz network, i.e., two-dimensional small-world network to be the underlying structure, on which we investigate the effect of randomly rewiring process on the cooperative behaviour. The network start with a two-dimensional square lattice with N nodes in which every node is connected to its K ($K = 4$) neighbours with periodic boundary conditions. Then randomly rewire each edge of the lattice with probability p excluding self-connections and duplicate edges. This progress generates $pNK/2$ long-range edges. By varying p one can closely observe the transition from order ($p = 0$) to randomness ($p = 1$).[4,12] Between $p = 0$ and $p = 1$, we can obtain SWNBRL. In the two-dimensional small-world networks, the rewiring probability p, one of main network parameters, governs the proportion of shortcuts.

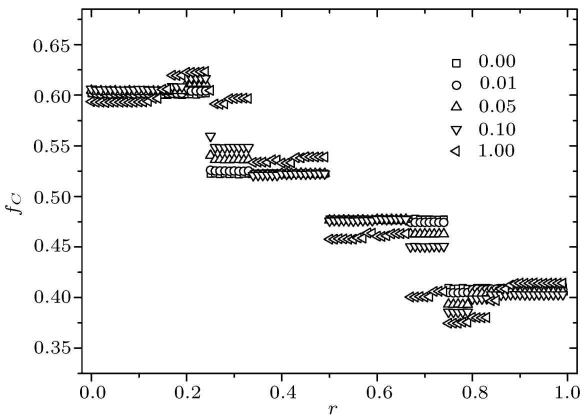

Fig. 1. Frequency of cooperation f_C as a function of the payoff parameter r for two-dimensional SWNBRL. We set $M = 2$ for different p values, as described by the insets. The network size is $N = 2500$.

Now we investigate the MBSG on two-dimensional small-world networks with a focus on the frequency of cooperators f_C, which is the most important quantity for characterizing cooperative behaviour in evolutionary games. It indicates the ratio of the number of cooperators and the total number of individuals. First, we study how f_C changes with the cost-to-reward ratio r. Simulations are carried out for a population of $N = 2500$ individuals located on nodes. Initially, the strategy of each individual is selected randomly, and also the memory information. Equilibrium frequencies of cooperation (f_C) are obtained by averaging from Monte-Carlo time step $t = 9000$ to $t = 10000$, where the system has reached a steady state. Each data point of simulation results is obtained by averaging over 50 different network realizations, with ten runs for each realization. Figure 1 show f_C as a function of r on SWNBRL, with $p = 0$ corresponding to regular lattices, $p = 1$ to pure random networks and other p values to small-world networks. We find that the MBSG on SWNBRL ($p \neq 0$ and $p \neq 1$) has some similar dynamical properties to the game on two-dimensional lattices ($p = 0$), including steps structure.[7] However, the cases of $p \neq 0$ have some remarkable differences with that of $p = 0$, i.e. there are four steps in regular four-neighbour lattices, but not exact four steps in SWNBRL anymore. Interestingly, some nonmonotonous phenomena occur, with a peak at the end of the first step and a minimum at the start of the last step, which turns prominent as p increases. One can find that the frequency of cooperation is promoted significantly around $r = 0.2$ and $r = 0.3$, and that it is inhibited around at $r = 0.7$ and $r = 0.8$.

Furthermore, to illustrate the effects of topological randomness p on cooperation for various r more precisely, we figure out how f_C changes with both p and r, as shown in Fig. 2. The gray level denotes the difference value of $(f_{C,p>0} - f_{C,p=0})$, where $f_{C,p>0}$ is the frequency of cooperation on small-world networks and $f_{C,p=0}$ is that on regular lattice. Some interesting phenomena appear: p enhances cooperation in the region of $r < 0.4$, but inhibits cooperation in the region of $r > 0.6$. Especially, f_C is promoted most in the region $p > 0.3$ around $r = 0.3$, in contrast, f_C is suppressed most in the region $p > 0.3$ around $r = 0.7$. There exists an optimal region of (r, p), where f_C is enhanced significantly. Similar results are also found in networks with different sizes ($N = 1600$ and $N = 10000$). The observed phenomena indicate that small-world effects play a nontrivial role on the MBSG in the existence of the optimal values of r resulting in the highest frequency of cooperation. The nonmonotonous behaviours of f_C depending on r with a peak in the middle range show that properly encouraging selfish actions may better promote the emergence and persistence of cooperation.

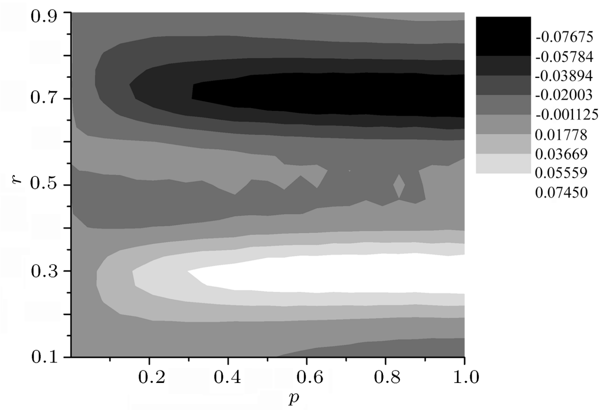

Fig. 2. Histogram of variation of f_C on small-world networks from that on regular lattice in the parameter plane of (r, p). The gray level denotes the difference values in each bin. M is set to be 2.

The steps structure on regular lattices has been explained in Ref. [7], in which a local stability equation is addressed: $m + (K - m)(1 - r_C) = (1 + r_C)m$, where m is the number of C neighbours of a given node and r_C is the critical point of different levels. This equation results in $r_C = (K - m)/K$. Considering all of the possible values of m in the 4-neighbour regular lattice, the values of r_C are 0.25, 0.5 and 0.75. For SWNBRL, however, there are more possible values of K and m. As a result, the step structure in SWNBRL is much more complex than that in regular network, as shown in Fig. 1.

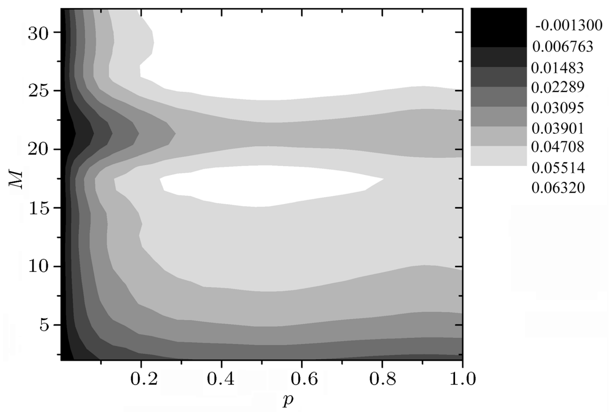

Fig. 3. Histogram of variation f_C on small-world networks from that on regular lattice in the parameter plane of (M, p). The gray level indicates the difference values in each bin. r is set to be 0.2.

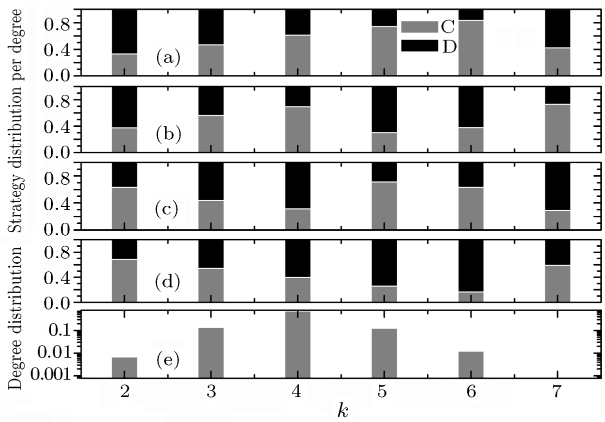

Fig. 4. Distribution of strategies on small-world lattices, which is obtained on SWNBRL with $p = 0.1$, when $M = 2$. Cooperators and defectors are denoted by gray bars and black bars, respectively. Each bar adds up to a total fraction of 1 per degree, the gray and black fractions being directly proportional to the relative percentage of the respective strategy for each degree of connectivity k. Here (a) $r = 0.15$, (b) $r = 0.21$, at which f_C peaks, (c) $r = 0.79$, where f_C reaches minimum value, (d) $r = 0.85$, (e) the gray bar factions are proportional to the ratio of number of nodes having degree of k and the total number N, versus each possible degree. All the simulation are obtained for network size $N = 2500$.

Next, we study the effects of memory length M on f_C on small-world networks. Generally, increasing p and M individually can promote frequency of cooperation, as in Fig. 3. Here $r = 0.2$ is selected here, for the reason that f_C increases and reaches a peak at about this value. Furthermore, it is shown that the increment of f_C becomes larger with the increasing p, but remains stable when $p > 0.3$. Moreover, on can also find that a larger value of M could stimulate f_C more remarkably. There exists optimal regions of $(16 < M < 18, 0.3 < p < 0.8)$ and $(M > 24, p > 0.2)$. However, the increasing value of M cannot promote f_C if p is set to be a value less than 0.2. We also investigate the phenomena in networks with different sizes ($N = 1600$ and $N = 10000$). As we found, there is no significant change of f_C in the parameter plane of (M, p). However, it is interesting that more optimal regions, like islands in ocean, appears in the parameter plane, namely turn up between islands in Fig. 3, if we consider the smaller network. On the other hand, in the larger one, there is only one island appearing in the region of $(M > 24, p > 0.3)$. The existence of an optimal region in parameter plane of (M, p) may shed some new light on the study of evolutionary games on networks.

In order to give an explanation for the nonmonotonous behaviours shown in Fig. 1, we study the degree k of cooperators and defectors depending on r. As shown in Figs. 4(a)–4(d), each bar is corresponding to an existent degree k in SWNBRL with $p = 0.1$, in which gray (black) faction is proportional to the relative percentage of cooperators (defectors). Also in Fig. 4(e), the gray bar factions show degree distribution, from which we can see that most nodes have degree of 4, and that only few nodes have degree of 7. Due to such few nodes having the highest degree, we ignore their influence on f_C. In comparison of Figs. 4(a) and 4(b), we find that the number of cooperators increases at low degree ($k = 2, 3, 4$) and decreases at high degree except $k = 7$ when $r = 0.21$ in comparison to $r = 0.15$. Note that most neighbours of high degree nodes are small degree ones. Hence, when high degree individuals switch to defection, partial small degree neighbours with defection strategy may change to cooperation to gain payoff $1 - r$ from these high degree defectors according to the payoff matrix of the snowdrift game. Otherwise, if the small degree defectors preserve their strategy, they gain nothing from high degree defectors. Due to the passive strategy switch of small degree defectors, the frequency of cooperation in Fig. 4(b) when $r = 0.21$ is higher than that in Fig. 4(a) when $r = 0.15$, thus a peak occurs in Fig. 1. It is simple to understand why f_C at $r = 0.85$ is larger than that at $r = 0.79$, in comparison of Figs. 4(c) and 4(d).

In conclusion, we have studied the memory-based snowdrift game on SWNBRL with tunable rewiring probability. The typical step structure of f_C is observed. In comparison of the results on SWNBRL with that on regular lattices, we find that randomly rewiring turns the original monotonous behaviour into nonmonotonous. Also we investigate the memory effect on the frequency of cooperation on SWNBRL. It is shown that optimal regions exist in both the parameter planes of (r, p) and (M, p). Moreover, the network size has little effect on the above results. The random rewiring effect on MBSG may draw some attention in the study of evolutionary games.

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