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Physica A 363 (2006) 492–500

PHYSICA A

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Simulation of evacuation processes using a multi-grid model for pedestrian dynamics

Weiguo Song[a,*], Xuan Xu[a], Bing-Hong Wang[a,b], Shunjiang Ni[a]

[a] *State Key Laboratory of Fire Science, University of Science and Technology of China, Hefei 230026, PR China*
[b] *Department of Modern Physics, University of Science and Technology of China, Hefei 230026, PR China*

Received 25 February 2005; received in revised form 20 July 2005
Available online 12 September 2005

Abstract

Introducing the force concept of a social force model into the lattice gas (LG) model, a new LG-based discrete model entitled "multi-grid model" is composed. In the new model, finer lattice is used; thus each pedestrian occupies multiple grids instead of one, and the rules of interactions among pedestrians or pedestrians and constructions are built. The interaction forces including extrusion, repulsion and friction are considered as passive factors for evacuation. The strength of the drift, or the intensity of the pedestrians to move toward the exit rapidly, is considered an active factor. A simple situation is studied in which pedestrians try to evacuate from a large room with only one door. The influences of interaction forces and drift on evacuation time are analyzed. The mutual restriction relation of the two factors in the course of evacuating is found.

Keywords: Multi-grid model; Passive factor; Active factor

1. Introduction

In the past several decades, vehicular traffic problems have attracted the attention of researchers, and methods of physics and modern computer science have been successfully used to study the problem [1–3]. Another type of many-body system, i.e. pedestrian flow, which is more difficult to describe in terms of simple models, has been studied in recent years [4–6]. Muramatsu et al. [7] have studied the counterflow of pedestrians within an underpass by using the lattice gas (LG) model of biased-random walkers. It has been found that the dynamic jamming transition occurs at a critical density. Nagatani et al. [8–11] have studied pedestrian flow under different conditions by using the same model and achieved valuable results. With the accumulation of the knowledge of the human behavior during a fire, study in human behavior modeling and evacuation modeling has improved. New methods and models in pedestrian dynamics have been introduced in the field of study [12]. Helbing et al. [13] have studied the dynamical features of escape panic using the social force model that perform features of escape panic such as arching and clogging of pedestrians at the exit. Kirchner et al.

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[*] Corresponding author.
E-mail address: wgsong@ustc.edu.cn (W. Song).

doi:10.1016/j.physa.2005.08.036

[14,15] have studied evacuation processes by using the cellular automata model based on the bionics and found that the evacuation time depends on the strength of the herding behavior, with minimal evacuation times for some intermediate values of the coupling constants.

As typical models in evacuation modeling, the LG model [16–19] and the social force model [13] can successfully simulate the most typical phenomena observed in pedestrian dynamics. They have their respective advantages in that the LG model is simple and has a high computational efficiency and the social force model is good at modeling the interactions among pedestrians.

Based on the classic LG model, we develop a new evacuation model entitled multi-grid model that uses finer lattice. In the new model, one pedestrian occupies multiple grids instead of one and overlapping of pedestrians is allowed. The interaction forces among pedestrians or those among pedestrians and constructions can be calculated quantitatively by the number and position of the overlapped grids, together with the model rules. In this paper, the study is focused on the influence of interactions on evacuation time and the mutual restriction relation between interactions and self-driven action. A typical scenario where many pedestrians try to escape from one door is simulated with the multi-grid model and the simulation results are discussed. The proposed model is expected to have some basic characteristics of both the LG model and the social force model.

2. Multi-grid model

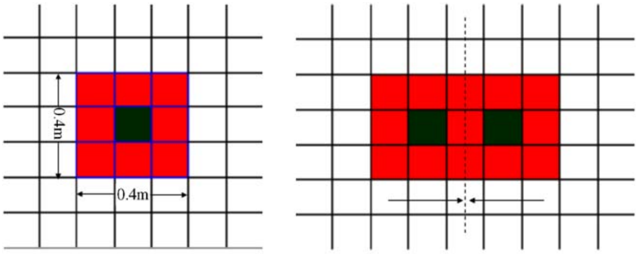
Let us consider the simplest parameters of the multi-grid model. In the model, space is represented with a two-dimensional lattice and each pedestrian occupies a foursquare district, the size of which is 40 cm × 40 cm. This is the typical space occupied by a pedestrian in a dense crowd. In the traditional LG model, each pedestrian is assumed to occupy only one grid with the same size as that of the pedestrian. To describe the interaction forces more conveniently and accurately, the multi-grid model uses a smaller grid, the size of which is approximately 13.3 cm × 13.3 cm or each pedestrian occupies 3 cm × 3 cm grids. As shown in Fig. 1(a), a black grid denotes the center of the pedestrian which is forbidden to be occupied by another pedestrian, and the red ones can be overlapped by other pedestrians. Considering that the compressibility of a pedestrian is limited, as many as three grids are allowed to be overlapped by other pedestrians. This is the "conflict rule" of the model. Fig. 1(b) is a possible overlapped state of two neighboring pedestrians. A pedestrian moves one grid in each time step with an average velocity of 1 m/s.

In the model, each pedestrian could select one of the eight possible directions with different transition probabilities and move to one of the neighboring grids (see Fig. 2).

We define $P_{i,j}$ as the transition probability that a pedestrian moves to the direction (i, j) and its value is given by

$$P_{i,j} = N\delta_{i,j}I_{i,j}\left(\frac{1-D}{\sum_{(i,j)}\delta_{i,j}} + D_{i,j} + \sum_{P} f_{i,j} + \sum_{W} f_{w}\right). \tag{1}$$

Here $\delta_{i,j}$ denotes the possibility whether the pedestrian can move to the direction (i, j), i.e., $\delta_{i,j} = 1$ if the direction is available and $\delta_{i,j} = 0$ if it is unavailable. The inertia $I_{i,j}$ represents the enhancement of the probability that the pedestrian keeps his previous movement direction. The value of $I_{i,j}$ is greater than 1 in the



(a) Size of one pedestrian (b) Two interactional pedestrians

Fig. 1. (a) Size of one pedestrian. (b) Two interactional pedestrians.

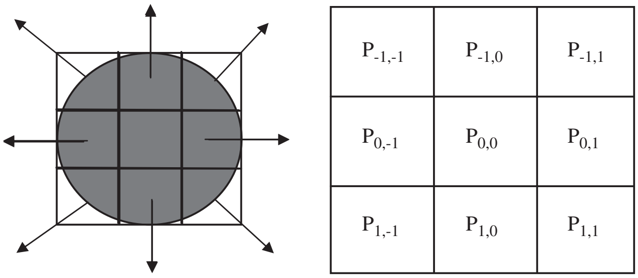


Fig. 2. Possible transitions for a particle and the associated probability $P_{i,j}$.

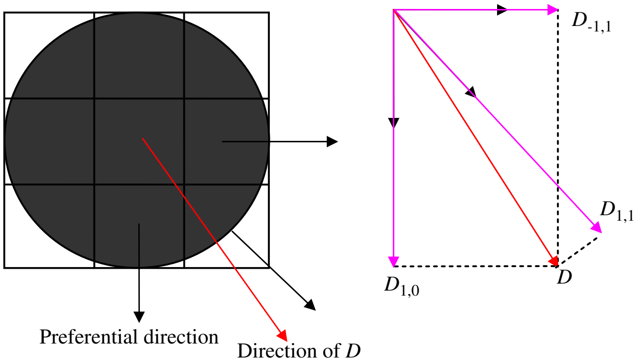


Fig. 3. Projection of preferential movement direction D.

previous movement direction and equal to 1 in other directions. N is a normalization factor to ensure $\sum_{(i,j)} p_{i,j} = 1$. The drift D here represents the intensity that a pedestrian rushes to the exit. The value of D is between 0 and 1, and the direction of it, or the preferential direction, points from the pedestrian to the exit. Because there are altogether eight possible moving directions of pedestrians, the preferential movement direction may deviate from any of the eight directions. Therefore, the preferential movement direction, i.e. D, is projected into three closest directions, as shown in formula (2) and Fig. 3.

$$D_{i,j} = D\cos\theta_{i,j}. \tag{2}$$

Here $D_{i,j}$ is the projection of D in direction (i, j), and $\theta_{i,j}$ is the angle between $D_{i,j}$ and D.

The interaction forces, including extrusion, repulsion and friction among pedestrians $\sum_P f_{i,j}$ and those among pedestrians and constructions $\sum_W f_w$, are considered as the passive factors that influence pedestrian movement. Once a grid of the pedestrian is overlapped with other pedestrians or constructions, he receives forces at this direction. The extrusion, repulsion and friction caused by the overlapped grid are represented with transition probabilities, as shown in Fig. 4(a) corresponding to formula (3a) and Fig. 4(b) corresponding to formula (3b).

$$\begin{array}{llll} f_{-1,-1} = -F, & f_{-1,0} = -\frac{\sqrt{2}}{2}\mu F, & f_{-1,1} = -\mu F, & f_{0,1} = \frac{\sqrt{2}}{2}F, \\ f_{1,1} = F, & f_{1,0} = \frac{\sqrt{2}}{2}F, & f_{1,-1} = -\mu F, & f_{0,-1} = -\frac{\sqrt{2}}{2}\mu F. \end{array} \tag{3a}$$

$$\begin{array}{llll} f_{-1,-1} = -\frac{\sqrt{2}}{2}\mu F, & f_{-1,0} = -F, & f_{-1,1} = -\frac{\sqrt{2}}{2}\mu F, & f_{0,1} = -\mu F, \\ f_{1,1} = \frac{\sqrt{2}}{2}F, & f_{1,0} = F, & f_{1,-1} = \frac{\sqrt{2}}{2}F, & f_{0,-1} = -\mu F. \end{array} \tag{3b}$$

Here F represents the average interaction force and $f_{i,j}$ the change of transition probabilities of all directions caused by the overlapped grid. Parameter μ is the friction coefficient. The positive/negative value indicates that

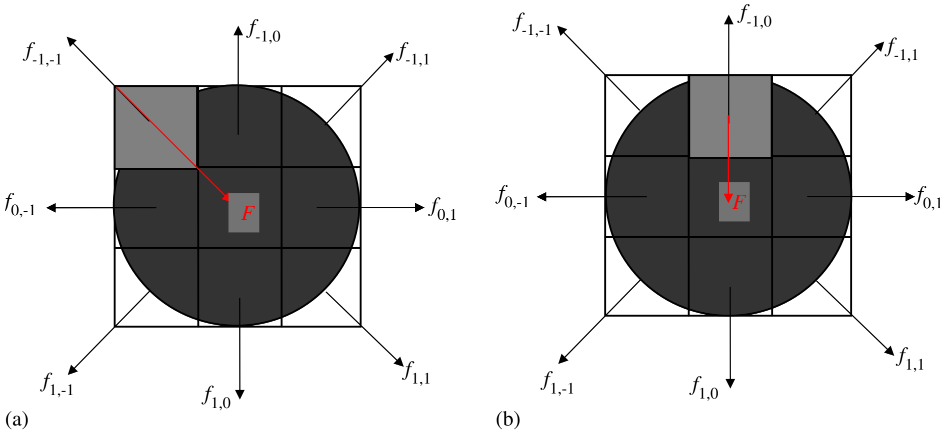


Fig. 4. (a) Interaction between two pedestrians through overlapped grid in upper-left corner. (b) Interaction between two pedestrians through overlapped grid in upper-middle corner.

forces exert positive/negative influences in the direction. If there are more grids overlapped, their effects should be superposed.

The update rules of the model are as follows:

(1) Calculating $p_{i,j}$ for each pedestrian.
(2) According to the value of $p_{i,j}$ in eight directions, a most possible movement direction is selected for each pedestrian.
(3) Updating the state of each pedestrian.

3. Results and discussions

The scenario of simulation is 200 pedestrians attempting to leave a one-door square room. The room size is 16 m × 16 m and the exit door size is W. Initially, 200 pedestrians are distributed randomly in the room. The inertia $I = 1.2$ and the average velocity of the pedestrians is 1 m/s. Other parameters are set as different values to study their impact on evacuation. The position of each pedestrian is updated with update rules.

Typical stages of the pedestrian dynamics are shown in Fig. 5. As pedestrians move to the exit, the arching behavior near the exit can be observed (see Fig. 5(b)). With the developing of arching behavior, the outward stream of pedestrians becomes discontinuous. The volume of each pedestrian is compressed and the crowd becomes denser and denser (see Fig. 5(c)). As more and more people leave the room, the size of arching decreases and finally disappears (see Fig. 5(d)).

Fig. 6 shows the relation between evacuation time and door size. With the increase of door width, evacuation time decreases. However, the change of evacuation time with door width is not linear. As the door size becomes larger and larger, the decrease in evacuation time is slowed down. When door width is small, i.e. smaller than 3 m, the relation is power-law-like (see inset of Fig. 6). With this curve, a door width can be determined as the number of pedestrians, room size and evacuation time is given.

Fig. 7 is a curve of evacuation times against drift. It shows that drift is the active factor for the evacuation. With the increase in drift, the evacuation time is reduced linearly. A large value of drift indicates that the pedestrian moves toward the door definitely instead of moving chaotically. Therefore, an ordered crowd evacuates faster than a disordered one.

Fig. 8 shows the change of evacuation time with the increase of friction coefficient. The increase of μ indicates the increase of friction. Because the friction in our model hinders pedestrians from moving to their preferential directions in the multi-grid model, the time of evacuation increases quickly with the increase of coefficient of friction μ.

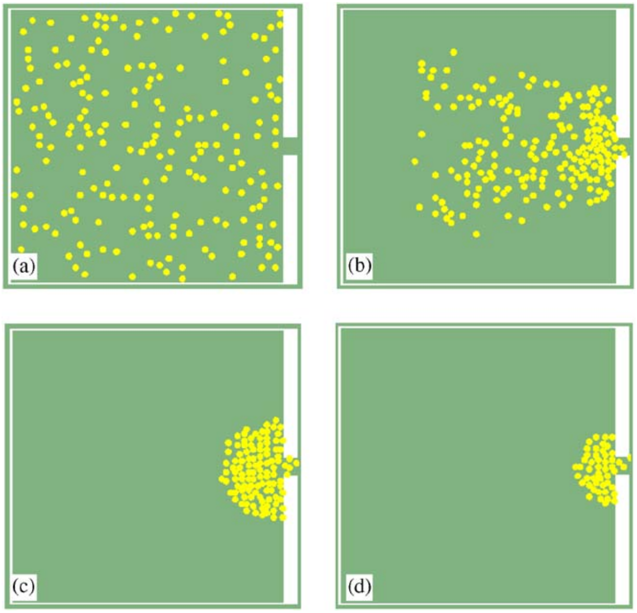


Fig. 5. Pedestrians leaving a room with one door. (a) Distribution of pedestrians. (b) Evacuation snapshot at 20 s. (c) Evacuation snapshot at 80 s. (d) Evacuation snapshot at 120 s.

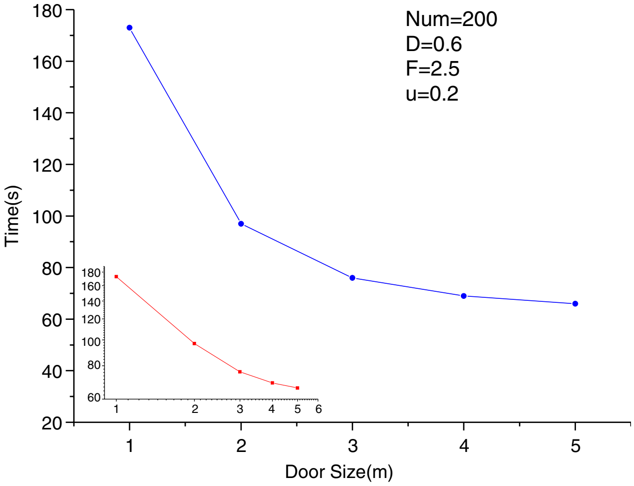


Fig. 6. Relation of evacuation time against door size.

Fig. 9 gives the relation between evacuation time and interaction force *F*. The interaction force *F* has three influences on the pedestrian. Firstly, pedestrians in the front obstruct the ones following. Secondly, pedestrians in the back push the ones in front toward the exit. Thirdly, there is friction between two pedestrians or between a pedestrian and the wall if they touch and overlap each other. The value of *F* can be regarded as the average interaction force among pedestrians or among pedestrians and construction. It represents the degree of crowding to a certain extent. Therefore, with the increase of *F*, the evacuation time increases.

In Fig. 10, the influences of the active factor *D* and the passive factor *F* on evacuation time are compared. With the increase of *F*, the slope of the linear fit line of the drift–time relation decreases. This indicates that the influence of *D* decreases. It is shown that the enhancement of the passive factor will slacken the influence of the

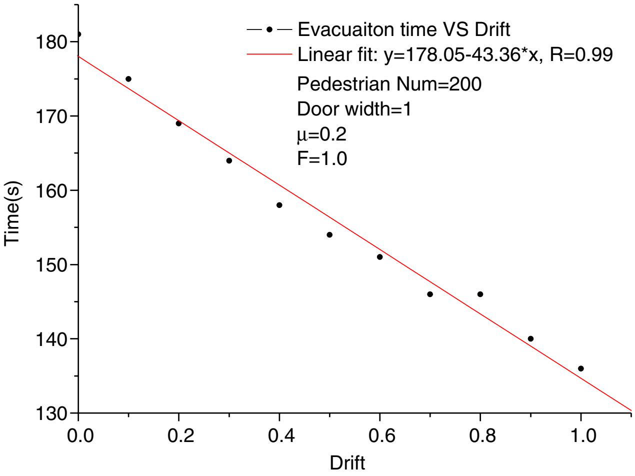


Fig. 7. Curve of evacuation times against drift.

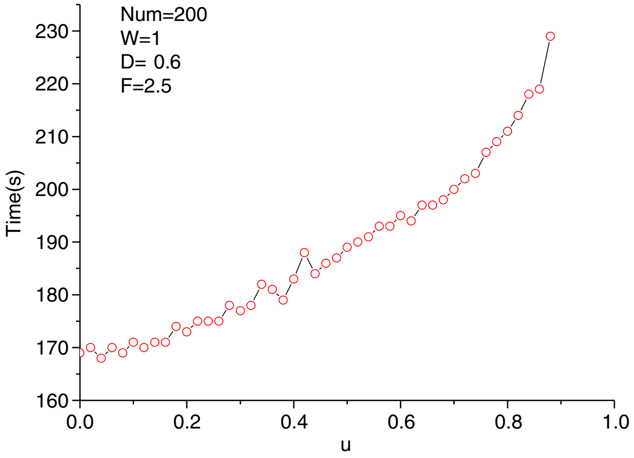


Fig. 8. Curve of evacuation times against coefficient of friction μ.

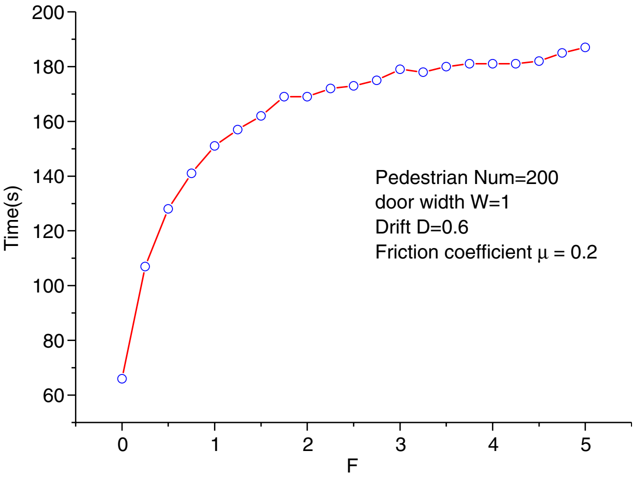


Fig. 9. Curve of evacuation times against interaction forces F.

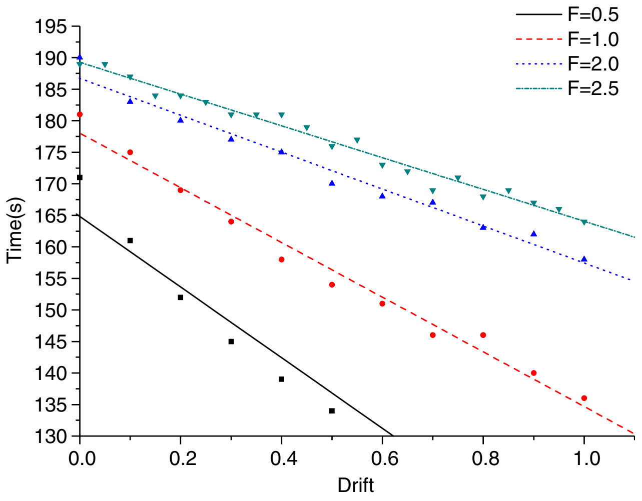


Fig. 10. Evacuation times as a function of drift for different F.

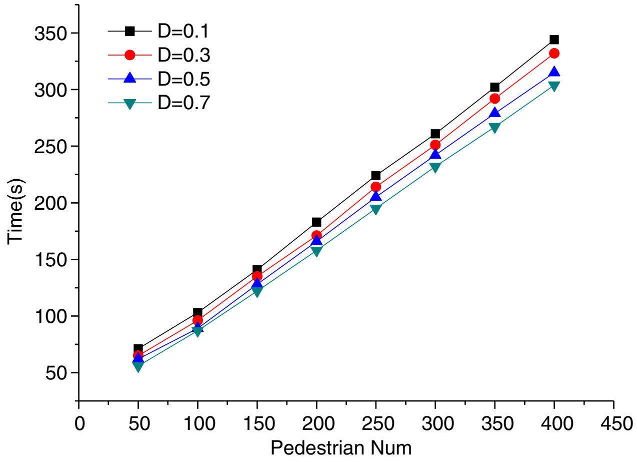


Fig. 11. Evacuation times as a function of the number of pedestrians for different D.

active factor. With the increase of F, that is, the increase of crowding degree, it becomes more and more difficult for pedestrians to control their movement, speed and direction, the pedestrians become more and more correlated with others, and the individual effort becomes weaker and weaker.

Figs. 11–13 show the influences of the active factor and the passive factor on evacuation time with the increase of the number of pedestrians. Fig. 11 shows that with the increase of D, the slope of the linear fit line of the time–number relation changes little. It indicates that the number of pedestrians has little influence on the value of drift in the current version of the multi-grid model. Figs. 12 and 13 show that the slope of the linear fit line of the time–number relation increases with the increase of the passive factors μ and F. It indicates that with the increase of the number of pedestrians, interaction forces increase, so that their influences on evacuation time become more and more obvious.

4. Summary

In summary, we propose a multi-grid evacuation model that uses a finer lattice in which the interaction forces among pedestrians and those between pedestrian and construction are quantified. During an evacuation, especially an emergency evacuation, pedestrians try to escape from the room as fast as possible,

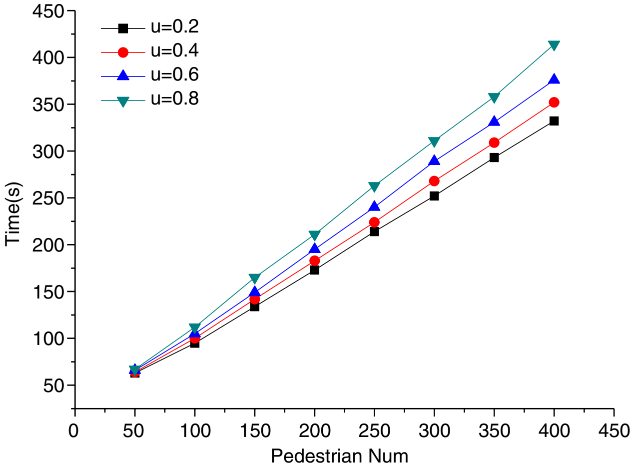


Fig. 12. Evacuation times as a function of the number of pedestrians for different μ.

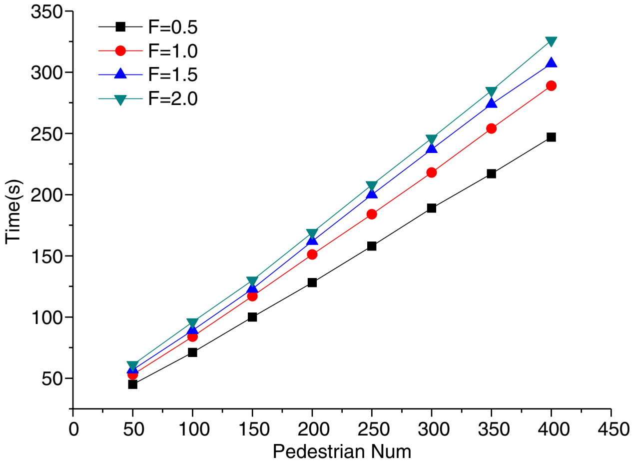


Fig. 13. Evacuation times as a function of the number of pedestrians for different F.

enhancing the interactions among pedestrians. Construction components such as walls and doors also have an impact on pedestrians during an evacuation. These self-driven actions and interactions decide the global behavior of the evacuating crowd. However, there is difficulty in quantifying pedestrian interactions as in a detailed as continuous model such as the social force model introduced by Helbing et al.[13]. Therefore, we introduce the force concept of the social force model into the multi-grid LG model. The interactions are represented with forces that are calculated with the number and position of overlapped grids. Thus, the repulsion and friction are modeled in a similar way with those of the social force model. The attraction of exit is represented with the strength of drift, which reflects the actual cooperative behaviors of pedestrians. The simulation results indicate that the model has some basic characteristics of the social force model, for example the arching and clogging behavior.

The arching and clogging behavior around the exit and the discontinuous outward stream of pedestrian flow reflect the impact of both repulsion and friction. With the increase of door width, the evacuation time decreases. The calculation results of the proposed model and social force model accord with each other.

The parameter D, i.e. drift, also explains the influence of order that has been discussed before [11,16,20]. In the ordered state or with a large value of drift, pedestrians tend to queue and wait instead of detour, so that the pedestrian flow near the exit and inside the room is in order. Hence the evacuation time is relatively small. In the disordered state or with a small value of drift, pedestrians detour and try to reach the exit faster, so that the arching and clogging near the exit become obvious and the evacuation time increases.

The influences of the friction coefficient μ and force F are similar. Evacuation time decreases with the increase of the two parameters. A large value of friction coefficient or force implies the high intensity of interactions among pedestrians. During a panic, pedestrians try to move faster, the interactions increase and thus the evacuation time increases.

Another characteristic is the high calculation speed. It will be useful for the simulation of safe evacuation during emergencies.

Acknowledgment

This study is supported by China National Key Basic Research Special Funds project (no. 2001CB409600), the National Natural Science Foundation of China (under Grant nos. 10472116, 50320120156, 70271070) and the Specialized Research Fund for the Doctoral Program of Higher Education (under Grant no. 20020358009). We would like to thank Prof. D. Helbing and Dr. I. Farkas for useful discussions on the social force model.

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