Topological Properties of Urban Public Traffic Networks in Chinese Top-Ten Biggest Cities *

LI Ping(李平)[1**], XIONG Xing(熊星)[1], QIAO Zhong-Liang(乔忠亮)[1], YUAN Gang-Qiang(袁刚强)[1], SUN Xing(孙兴)[1], WANG Bing-Hong(汪秉宏)[2]

[1] *Department of Basic Sciences, Nanjing Institute of Technology, Nanjing 210013*

[2] *Department of Modern Physics and Nonlinear Science Center, University of Science and Technology of China, Hefei 230026*

(Received 18 August 2006)

We investigate the topological characteristics of complex networks as exemplified by the urban public traffic network (UPTN) in Chinese top-ten biggest cities. It is found that the UPTNs have small world behaviour, by the examination of their topological parameters. The quantitative analysis of the transport efficiency of the UPTNs reveals their higher local efficiency E_l and lower global efficiency E_g, which coincide well with the status quo of those Chinese cities still at their developing stage. Furthermore, the topological properties of efficiency in the UPTNs are also examined, and the findings indicate that, on the one hand, the UPTNs show robustness to random attacks and frangibility to malicious attacks on a global scale; on the other hand, the interrelation between UPTN efficiency and network motifs deserves our attention. The motifs which interrelate the UPTN efficiency are always triangular-formed patterns, e.g. motifs ID 238, ID 174 and ID 102, etc.

PACS: 89.70.+c, 05.90.+m, 89.40.+k, 89.75.Hc

Nowadays, urban public traffic is one of the most important ingredients of modern city life. To find a comprehensive solution to the urban public traffic problem, we should adopt a multi-disciplinary approach.

Recently, many natural and man-made systems are characterized by complex networks with topological pedestals. The complex network provides an approach to complex systems with many interacting units where the details of the interactions are of less importance, while the essentials are focused on Refs. [1–3]. The complex network was recently introduced in statistical physics, as a basic tool to help us to understand a variety of phenomena in interdisciplinary fields. This approach has proven to be extremely useful in many fields such as the internet, biology, economics, physics and sociology.[4,5] Obviously, it is reasonable to describe an urban public traffic system as a complex network. During the past few years, transport systems have been investigated with the approach of network topological statistics.[6−11]

The urban public traffic system can naturally be represented by a weighted network, termed an urban public traffic network (UPTN), in which the vertices are bus-stations and the edges are non-station lines between any two bus-stations. If bus-stations i and j have multiple lines, the magnitude $w_{ij}$of the number of the multiple lines is assigned to the edge of vertices i and j as its weight. In the UPTNs of Chinese top-ten biggest cities, the number N of nodes ranges from 808 to 3996.

An UPTN must have a corresponding adjacency matrix $\{w_{ij}\}$, which is a real asymmetric matrix. All topological characteristics can be extracted from the adjacency matrix. For example, we can obtain some topological characteristic statistical parameters in the UPTN, i.e. mean degrees of nodes $\langle k\rangle$, mean strength of nodes $\langle s\rangle$, mean clustering coefficient $\langle C\rangle$, mean characteristic path length $\langle l\rangle$, betweenness centricity BC (see Table 1). Both $\langle C\rangle$ and $\langle l\rangle$, calculated from empirical data, can be compared with[12]

$$\langle C_R\rangle \approx \frac{\langle k\rangle}{N}, \quad \langle l_R\rangle \approx \frac{\ln N}{\ln\langle k\rangle}, \tag{1}$$

corresponding to random networks, and

$$\langle C_r\rangle \approx \frac{3}{4}\frac{(\langle k\rangle-2)}{(\langle k\rangle-1)}, \quad \langle l_r\rangle \approx \frac{N}{2\langle k\rangle}, \tag{2}$$

corresponding to regular networks, respectively (also see Table 1).

The UPTNs are neither completely ordered nor completely random, but rather exhibit important properties. From Table 1 we find that $\langle l\rangle$ in UPTNs is always less than $\langle l_r\rangle$ corresponding to regular networks, and $\langle C\rangle$ in UPTNs is always greater

* Supported by the National Basic Research Program of China under Grant No 2006CB705500, the National Natural Science Foundation of China under Grant Nos 10635040, 10532060, 10472116 and 70571074, the Special Research Funds for Theoretical Physics Frontier Problems from the National Natural Science Foundation of China under Grant Nos 10547004 and A0524701, the Science and Technology Foundation of Nanjing Institute of Technology under Grant No KXJ06048, and the Foundation for Students of Nanjing Institute of Technology.

** To whom correspondence should be addressed. Email: lp14@163.com

than $\langle C_R \rangle$ corresponding to random networks. We can conclude that the UPTNs exhibit the small world behaviour.[12,14] Some properties of the UPTNs can be embodied by small world network.

Table 1. Topological characteristics of UPTN in Chinese top-ten biggest cities.[13]

City	N	$\langle k \rangle$	$\langle s \rangle$	$\langle C \rangle$	$\langle l \rangle$	BC	$\langle C_R \rangle$	$\langle C_r \rangle$	$\langle l_R \rangle$	$\langle l_r \rangle$
Chengdu	1498	6.29	10.2	0.0912	13.00	0.00067	0.0042	0.608	3.98	119.1
Guangzhou	1035	7.10	16.2	0.1439	11.00	0.00032	0.0077	0.643	3.34	64.7
Hangzhou	808	6.36	10.7	0.1194	28.00	0.00145	0.0079	0.610	3.62	63.5
Shanghai	2037	9.22	17.2	0.2170	16.10	0.00049	0.0045	0.659	3.43	110.6
Shenyang	1095	6.17	24.2	0.1399	20.60	0.00091	0.0056	0.605	3.84	88.7
Shenzhen	953	5.73	11.0	0.1217	25.36	0.00105	0.0060	0.591	3.93	83.2
Tianjin	1641	7.02	14.9	0.1350	16.00	0.00061	0.0043	0.625	3.80	116.8
Wuhan	811	6.35	14.1	0.1102	13.70	0.00124	0.0078	0.610	3.62	63.9
Nanjing	1542	5.86	11.2	0.1110	17.00	0.00064	0.0038	0.596	4.15	131.5
Beijing	3996	6.35	15.1	0.1518	16.79	0.00025	0.0016	0.614	4.42	305.9

The structure and connectivity properties of complex networks contribute greatly to their functions and efficiency. It is therefore useful to ask whether these properties can be exploited to enhance the performance and efficiency of the UPTNs. A question arises naturally concerning how much the topological structures of the UPTNs influence the efficiency. The concept of measuring of efficiency of informational exchange by the network is both global and local efficiency.[15−17]

We have known that the UPTN always corresponds a weighted matrix $\{w_{ij}\}$, in which the weight w_{ij} is the number of multiple lines from vertices i and j. From vertex i to vertex j by an alternating sequence of vertices and edges, if the sum

$$\frac{1}{w_{ik}}+\frac{1}{w_{kl}}+\frac{1}{w_{lm}}+\cdots+\frac{1}{w_{nj}} \tag{3}$$

is minimum, then the path is defined as the shortest one. The shortest path length d_{ij} is defined as the number of alternately traversed edges. Then we have a corresponding shortest path length matrix $\{d_{ij}\}$.

The efficiency in the communication between two vertices i and j is calculated as the inverse of the shortest path length d_{ij} between two vertices:

$$e_{ij}=\frac{1}{d_{ij}}. \tag{4}$$

Then the global efficiency of the network is calculated as the average e_{ij} over all pairs of nodes[15,16]

$$E_g=\frac{1}{N(N-1)}\sum_i\sum_{j\neq i}e_{ij}=\frac{1}{N(N-1)}\sum_i\sum_{j\neq i}\frac{1}{d_{ij}}, \tag{5}$$

where d_{ij} is the length of the shortest path between i and j, i.e., the number of edges in the shortest path connecting the two, and the factor N $(N-1)$ is the number of complete pairs of vertices.

The local efficiency can be defined as an average efficiency $E(G_i)$ of the local sub-networks of the first neighbours G_i of each vertex i $(i \in G_i)$:[15,16]

$$E_l=\frac{1}{N}\sum_i E(G_i). \tag{6}$$

In the limit case of completely connected networks, i.e. any two vertices i and j are always the first neighbours, $d_{ij}\equiv 1$, we have $E_g \equiv E_l \equiv 1$.

Figure 1 presents the results of both global and local efficiencies of the UPTN in Chinese top-ten biggest cities. Meanwhile, we also compare the respective values with the corresponding random networks.

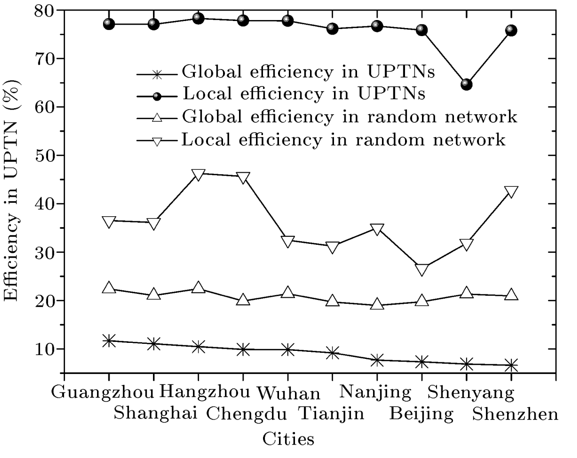


Fig. 1. Global efficiency and local efficiency in both the UPTNs of Chinese top-ten biggest cities and the corresponding random networks over ten independent simulation runs.

Figure 1 also shows that every UPTN in Chinese top-ten biggest cities is a very efficiency transportation system on a local scale, but not at the global level. The case, on the one hand, can be explained by the fact that the UPTNs are always constructed at local level; on the other hand, the UPTNs need to be adjusted in the expansion process of cities. Then the UPTNs on global scale would transform into local scale with the rapid expansion of cities in China. These coincide well with the fact that Chinese cities are at their developing stages. However, during the development, a balance must be kept between the global and local efficiency for Chinese UPTN.

It is interesting to study in detail the relation between efficiency and fault tolerance or substructure of the UPTN. We illustrate how these global and local efficiency behaviours in a dynamics environment vary, where some topological significant vertices could be eliminated at a certain percentage. A key issue in the characterization of networks is the identification of the most central nodes, i.e. hubs in networks. We have proposed a series of topological measures, i.e. degree k, strength s, and betweenness centricity BC, which can be used to identify important central vertices. According to the rank in terms of these topological measure magnitudes, a few nodes can be eliminated at a certain percentage. In this case, we can analyse the relation between the elimination rate and efficiency of UPTN.

Figure 2 indicates that significant elimination nodes with higher degree, strength or betweennes centrality have serious repercussions on global efficiency but not on local efficiency of both UPTNs and random networks. The properties can be explained by the fact that since the local efficiency E_l exhibits how the system tolerates faults and how efficient is the communication between the first neighbours of i when i is removed, the local efficiency E_l is almost constant magnitude in both UPTNs and random networks. However, for global efficiency E_g, degree of node more dramatically influences the UPTN properties than other topological measures.

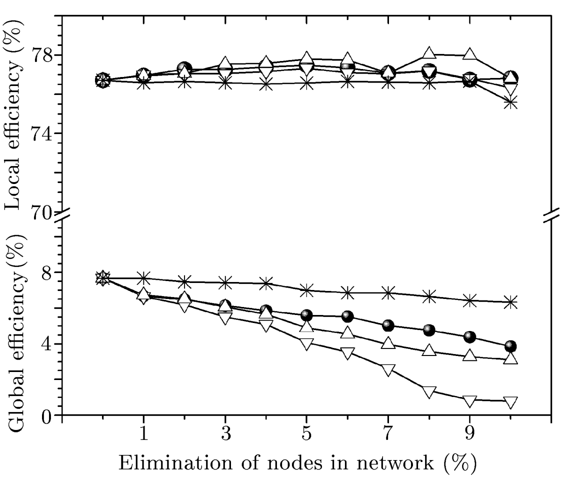
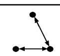
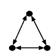
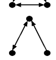
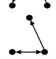
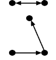
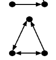
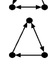
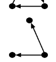
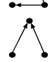
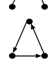
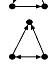
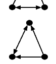
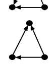


Fig. 2. Global and local efficiencies in the UPTN of Nanjing versus elimination rate of node by descending order of degree k (dow-triangle), strength s (circle), betweenness centricity (up-triangle), or random removal(asterisk) over ten independent simulation runs.

It is also noticed that the existence of some key sub-graphs (motifs) would dramatically influence the dynamics properties of complex networks. Motifs, a concept recently introduced to denote those sub-graphs, display a significantly frequency higher than that expected for random networks.[18,19] Frequently occurring motifs have been related to the functionality of complex networks. Recent studies have shown that the motif connectivity properties of networks contribute greatly to their functions and efficiency. By using FANMOD,[20] we obtain the occurring frequencies of all 13 patterns of directed three-nodes connected motifs in both original UPTN of Shanghai and the corresponding random networks (see Table 2).

Table 2. The first column, Motif, gives all 13 patterns of directed three-nodes connected motifs; the second column, ID (identification), is characteristic integers for the motif. It is obtained by interpreting the corresponding 0-or-1-adjacency matrix as a binary integer; the third column, frequency (original), denotes the frequency with which a motif occurs in the original network; the last column is the mean frequency with which the motif occurs in the corresponding random networks. The original network is the UPTN of Shanghai.

Motif	ID	Frequency (original)	Mean frequency (random)
	78	90.75%	97.83%
	238	6.91%	0.00%
	164	1.08%	1.05%
	14	0.98%	0.96%
	12	0.15%	0.14%
	174	0.10%	0.01%
	102	0.02%	0.00%
	6	0.01%	0.01%
	36	0.01%	0.01%
	140	0.00%	0.00%
	46	0.00%	0.00%
	166	0.00%	0.00%
	38	0.00%	0.00%

Not only in the UPTN of Shanghai, but also in those of other cities, we find that the motifs, which display significantly frequency higher than that expected for random networks, are always triangular-formed patterns, e.g. motifs ID 238, ID 174 and ID 102, etc.

According the eliminated percentage of important nodes that have highest degrees, i.e. from 1% to 10%, we obtain the change of occurrence rate of motif ID238 (see Fig. 3). The occurrence rate of motif ID238 is highly sensitive to node degree. This fact indicates that motifs are not distributed uniformly within the UPTN but tend to aggregate around the significant nodes, i.e. hubs.

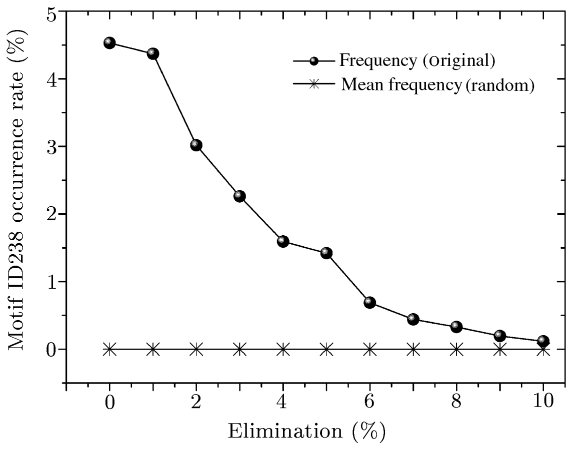


Fig. 3. Motif ID238 occurrence rate versus elimination percentage of key nodes that have highest degrees in the UPTN of Nanjing.

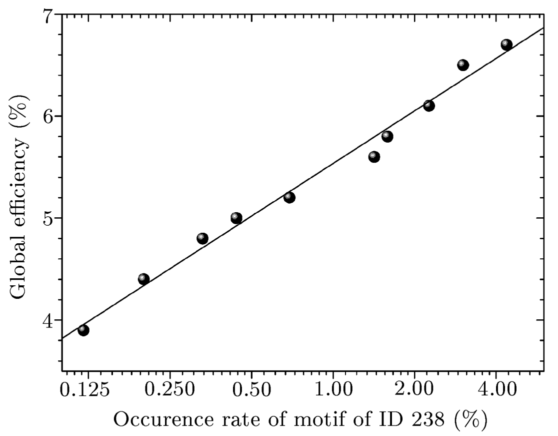


Fig. 4. Global efficiency in UPTN of Nanjing versus motif ID238 occurrence rate.

Then we discuss the relation between the global efficiency and motif in UPTNs. The main purpose of this approach focuses on how types of motif affect the global efficiency of UPTN so as to capture essential features of efficiency in UPTN. Figure 4 illustrates the relation between the global efficiency of UPTN in Nanjing and occurrence rate of motif ID238. They are so closely related that UPTN large-scale topology can be uncovered from the inspection of the significant motifs, which have also been found in Table 2. The statistical characterization of the UPTN local topology allows us to determine its statistical features of the global scale.[21]

The UPTNs have different structures and functions. However, seen from complex network topology, certain features are common to those UPTNs. They all exhibit the small world phenomenon, and are very efficient transportation systems on the local scale but not at global level. Thus they can better resist accidental damage. Both functions and efficiencies have notable difference for different three-node motifs. The stability property on global scale but not on local scale to perturbations is highly correlated with the relative abundance of triangular-formed motifs, e.g. motif ID238. This topological approach provides a new perspective and adds to our understanding about UPTN.

References

[1] Barabási A L and Albert R 1999 *Science* **286** 509
[2] Barabási, A L, Albert R and Jeong H 1999 *Physica* A **272** 173
[3] Newma M E J and Watts D J 1999 *Phys. Rev.* E **60** 7332
[4] Albert R and Barabási A L 2002 *Rev. Mod. Phys* **74** 47
[5] Boccaletti S, Latora V, Moreno Y, Chavez M and Hwang D U 2006 *Phys. Rep.* **424** 175
[6] Latora V and Marchiori M 2002 *Physica* A **314** 109
[7] Wu J J, Gao Z Y, Sun H J and Huang H J 2004 *Mod. Phys. Lett.* B **18** 1043
[8] Sienkiewicz J and Holyst J A 2005 *Phys. Rev.* E **72** 046127
[9] Wang R and Cai X 2005 *Chin. Phys. Lett.* **22** 2715
[10] Dall'Asta L, Barrat A, Barthélemy M and Vespignani A 2006 *J. Stat. Mech.* **4** p04006
[11] Wu J J, Gao Z Y and Sun H J 2006 *Europhys. Lett.* **74** 560
[12] Watts D J 2003 *Small Worlds* (Princeton, NJ: Princeton University Press)
[13] www.8684.com
[14] Watts D J and Strgatz S H 1998 *Nature* **393** 440
[15] Latora V and Marchiori M 2001 *Phys. Rev. Lett.* **87** 198701
[16] Latora V and Marchiori M 2003*Eur. Phys. J.* B **32** 249
[17] Vragovic I, Louis E and Dìaz-Guilera A 2005 *Phys. Rev.* E **71** 036122
[18] Milo R, Shen-Orr S, Itzkovitz S, Kashtan N, Chklovskii D and Alon U 2002 *Science* **298** 824
[19] Milo R, Itzkovitz S, Kashtan N, Levitt R, Shen-Orr S, Ayzenshtat I, Sheffer M and Alon U 2004 *Science* **303** 1538
[20] Wernicke S and Rasche F 2006 *Bioinformatics* **22** 1152
[21] Vázquez A, Dobrin R, Sergi D, Eckmann J P, Oltvai Z N and Barabási A L 2004 *Proc. Nat. Acad. Sci.* **101** 17940