Scaling behavior of an artificial traffic model on scale-free networks

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Abstract

In this Letter, we investigate an artificial traffic model on scale-free networks. Instead of using the routing strategy of the shortest path, a generalized routing algorithm is introduced to improve the transportation throughput, which is measured by the value of the critical point disjoining the free-flow phase and the congested phase. By using the detrended fluctuation analysis, we found that the traffic rate fluctuation near the critical point exhibits the $1/f$-type scaling in the power spectrum. The simulation results agree very well with the empirical data, thus the present model may contribute to the understanding of the underlying mechanism of network traffics.
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1. Introduction

Complex networks can be used to describe a wide range of systems from nature to society, thus there has been a quickly growing interest in this area since the discoveries on small-world phenomena [1] and scale-free properties [2]. Due to the increasing importance of large communication networks upon which our society survives, the dynamical processes taking place upon the underlying structures such as traffics of information flow have draw more and more attentions from the physical and engineering communities. Previous studies on understanding and controlling traffic congestion on network have a basic assumption that the network has a homogeneous structure [3–6]. However, many real communication networks, including the Internet [7] and WWW [8], display the scale-free property. Therefore it is of great interest to explore how this highly heterogeneous topology affects the traffic dynamics, which have brought a number of research works in recent years (see the review paper [9,10] and the references therein).

To improve the transportation efficiency on complex networks, Guimerá et al. presented a formalism that can deal simultaneously with the searching and traffic dynamics in parallel transportation systems [11]. This formalism can be used to optimize network structure under a local search algorithm through the knowledge of the global information of whole network. By using a global and dynamical searching algorithm aimed at the shortest paths, Zhao et al. provided the theoretical estimates of the communication capacity [12]. Holme proposed a much intelligent routing protocol in which the packet can detour at obstacle thus guarantee much better performance than the traditional routing strategy which just waits at obstacle [13]. Since the global information is usually unavailable in large-scale networks, Tadić et al. proposed a traffic model on a mimic WWW network [14] wherein only the local information, is available. This work highlighted the relationship of global statistical properties and microscopic density fluctuations [15,16]. In addition, Yin et al. [17,18] investigated how the local routing protocol affects the traffic condition. The effect of non-uniform delivering and generating rate [19], the performance of adaptive routing strategy [20], and the routing strategies for the Internet [21] as well as disordered networks [22] are also studied.

Recent empirical studies on communication networks have found pervasive evidence of some surprising scaling proper-

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ties. One example of such discoveries is that the traffic rates of a given link in the Internet (i.e., the number of packets or bytes traversing a given link per time unit) exhibit the self-similar (or fractal-like) scaling, and the multifractal scaling is also found over small time scales [6,23–26]. These empirical studies describe pertinent statistical characteristics of temporal dynamics of measured traffic rate process and provide ample evidence that these traces are consistent with long-range correlated behavior. Faloutsos et al. concerned the autonomous system (AS) of the Internet and pointed out for the first time that the degree distribution of snapshots in the AS level follows a heavy-tailed function that can be reasonably approximated by power-law forms [27]. Following these pioneering works, Percacci et al. found that the round trip time (RTT) or Internet delay time can also be characterized by a slow power-law decay [28]. This finding implies that network inter-connectivity topology structure of Internet and transportation delay both have a highly heterogeneous characteristic. Thus, the assumption of Poisson process, which was a major part of previous traffic model, has clearly lost its validity. This is also the very reason why in this article we use scale-free networks for simulations. Furthermore, the observation of a phase transition between the free-flow phase and the congested phase in the Internet traffic is demonstrated by Takayasu et al. through both the RTT experiment [29,30] and packet flow fluctuation analysis [31,32]. However, the $1/f$-type scaling is only detected near the critical state [29–32].

In this Letter, we investigate a traffic model on scale-free networks, in which packets are routed according to the global topological information with a single tunable parameter β. Instead of using the routing strategy based on the shortest paths, we give a generalized routing algorithm to find the so-called *efficient path* to improve the transportation efficiency that is measured by the value of the critical point disjoining the free-flow and the congested phases. Furthermore, by using the detrended fluctuation analysis, we investigate the statistic properties of traffic rate process (i.e., depicted by the number of packets per time unit) based on this traffic model, which is in good accordance with the empirical data.

2. Model

In this Letter, we treat all the nodes as both hosts and routers [11,33]. The model is described as follows: at each time step, there are R packets generated in the system, with randomly chosen sources and destinations. It is assumed that all the routers have the same capabilities in delivering and handling information packets, that is, at each time step all the nodes can deliver at most C packets one step toward their destinations according to the routing strategy. We set $C = 1$ for simplicity. A packet, once reaching its destination, is removed from the system. We are most interested in the critical value R_c where a phase transition takes place from free flow to congested traffic. This critical value can best reflect the maximum capability of a system handling its traffic. In particular, for $R < R_c$, the numbers of created and delivered packets are balanced, leading to a steady free traffic flow. For $R > R_c$, traffic congestion occurs as the number of accumulated packets increases with time, simply for that the capacities of nodes for delivering packets are limited. To characterize the phase transition, we use the following order parameter

$$H(R) = \lim_{t \to \infty} \frac{C}{R} \frac{\langle \Delta W \rangle}{\Delta t}, \tag{1}$$

where $\Delta W = W(t + \Delta t) - W(t)$, with $\langle \cdots \rangle$ indicating average over time windows of width Δt, and $W(t)$ is the total number of packets in the network at time t. Clearly, H equals zero in the free-flow state, and becomes positive when R exceeds R_c.

Consider the routing under global protocol [34], where the whole structural information is available, and the fixed routing algorithm is the most widely used one for its obvious advantages in economical and technical costs [35,36]. Actually, the path with shortest length is not necessarily the quickest way, considering the presence of possible traffic congestion and waiting time along the shortest path. Obviously, nodes with larger degree are more likely to bear traffic congestion, thus a packet will by average spend more waiting time to pass through a high-degree node. All too often, bypassing those high-degree nodes, a packet may reach its destination quicker than taking the shortest path. In order to find the optimal routing strategy, we define the *efficient path* [34]. For any path between nodes i and j as $P(i \to j) := i \equiv x_0, x_1, \ldots, x_{n-1}, x_n \equiv j$, denote

$$L\big(P(i \to j) : \beta\big) = \sum_{i=0}^{n-1} k(x_i)^\beta, \tag{2}$$

where $k(x_i)$ denotes the degree of the node x_i, and β is a tunable parameter. The efficient path between i and j is corresponding to the route that makes the sum $L(P(i \to j) : \beta)$ minimum. Obviously, $L_{\min}(\beta = 0)$ recovers the traditionally shortest path length. As for any pair of source and destination, there may be several efficient paths between them. We randomly choose one of them and put it into the fixed routing table which is followed by all the information packets.

3. Simulation and analysis

Some simulation and analytic results are presented in this section. Without particular statement, all the simulations are based on the Barabási–Albert (BA) networks [2] with average degree $\langle k \rangle = 4$.

3.1. Phase transition

The packets handling and delivering capacity of the whole network is an index of performance in traffic system, which can be measured by the critical value R_c. At the critical point, a continuous transition will be observed from the free-flow phase to the congested phase. Fig. 1 shows the order parameter H versus R with different β, in which the obvious phase transitions occur. For different β, it is easy to find that the capacities of systems are much different.
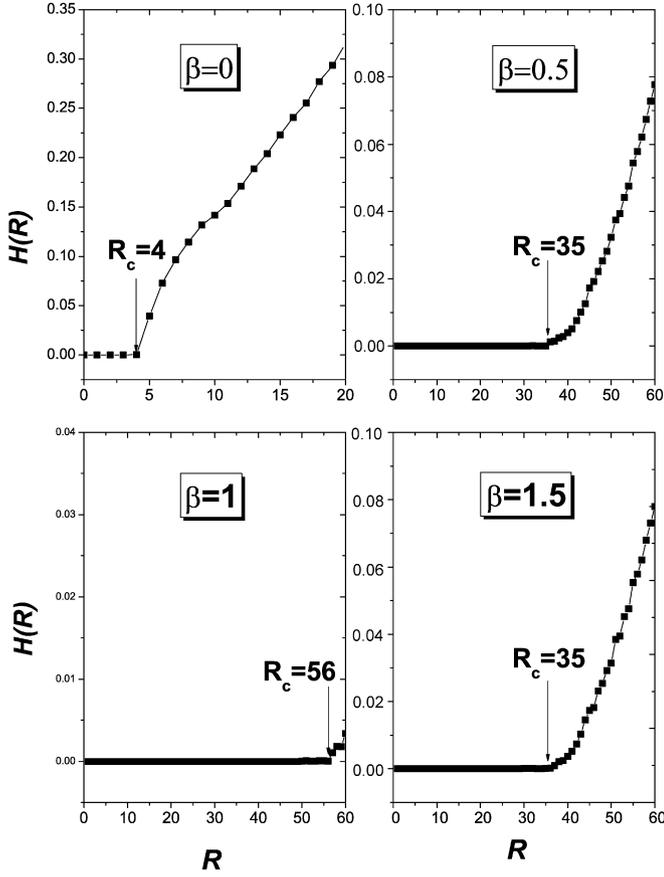
Fig. 1. The order parameter H vs. R for BA networks with average degree $\langle k \rangle = 4$ and of size $N = 1500$. The estimative values of critical point R_c of phase transition are 4, 35, 56 and 35 for $\beta = 0$, 0.5, 1 and 1.5, respectively.

3.2. Optimal routing strategy

We select packet routing strategy through a tunable parameter β, and the capacities for different β are much different. A natural question arises: Which value of β will lead to the maximal capacity of the network. In Fig. 2, we report the simulation results for the critical value R_c as a function of β on BA networks with the size $N = 1225$ and $N = 1500$, which demonstrate that the optimal routing strategy is corresponding to $\beta = 1$ and the size of BA network does not affect the optimal value. In comparison with the traditional routing strategy (i.e., $\beta = 0$), the capacity R_c of the whole network is greatly improved more than 10 times without any increase in algorithmic complexity.

By extending the concept of *betweenness centrality* [37,38] to *efficient betweenness centrality*,[1] the analytic results can be obtained according to the Little's law. (One can find the concept of efficient betweenness and the details about the analytical approach in Refs. [11,12,34].) The analytical results are also shown in Fig. 2, which agree very well with the simulations.

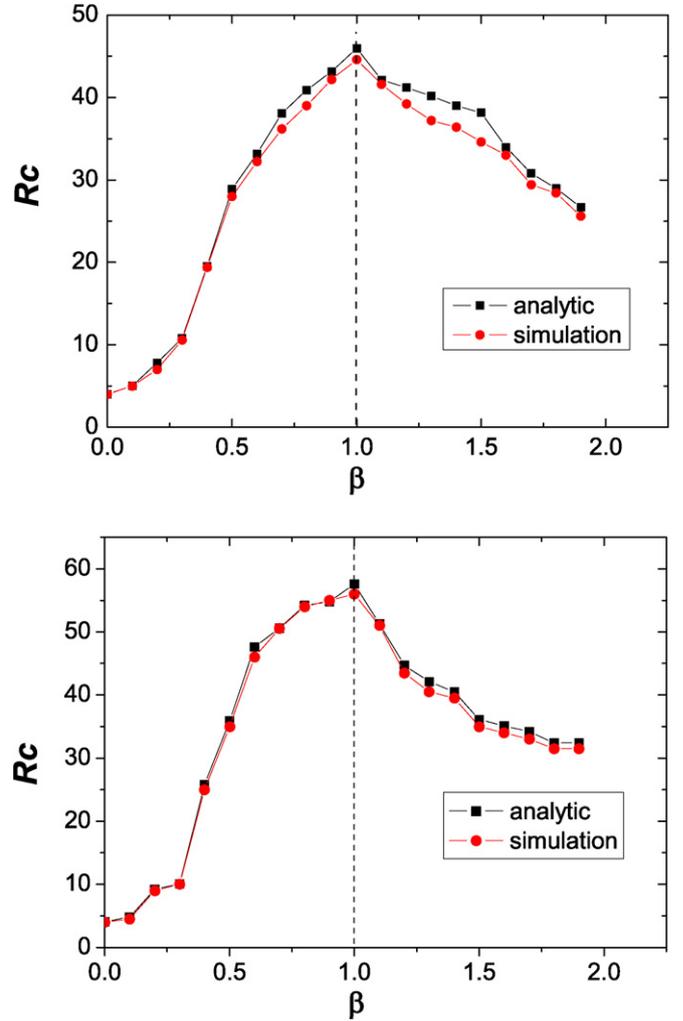[1] Using the efficient paths instead of the shortest paths in the definition of betweenness centrality.

Fig. 2. The critical R_c vs. β for BA network with size $N = 1225$ (up panel) and $N = 1500$ (down panel). Both the simulation (black squares) and analysis (red circles) demonstrate that the maximal R_c corresponds to $\beta \approx 1.0$. The results are the average over 10 independent runs. (For interpretation of the references to colour in this figure legend the reader is referred to the web version of this Letter.)

4. Detrended fluctuation analysis on the scaling behavior of the traffic rate process

4.1. The critical phenomena of the traffic rate process

The empirical studies on communication networks confirm the existence of critical phenomena in traffic rate process [31,32]. The congestion duration length distribution obeys a power-law form with an approximate exponent -1 in critical state, which implies the self-similarity scaling exponent (Hurst exponent) $H^u \approx 1$. Actually, $H^u < 1/2$ stands for an anti-persistent long-range correlated process in which a step in one direction is preferentially followed by a reversal of direction, while $H^u > 1/2$ is interpreted as a persistent long-range correlated process in which a step in one direction is preferentially followed by another step in the same direction. A value of $H^u = 1/2$ is interpreted as the ordinary diffusion (random walk), in which each step is independent of its preceding one.
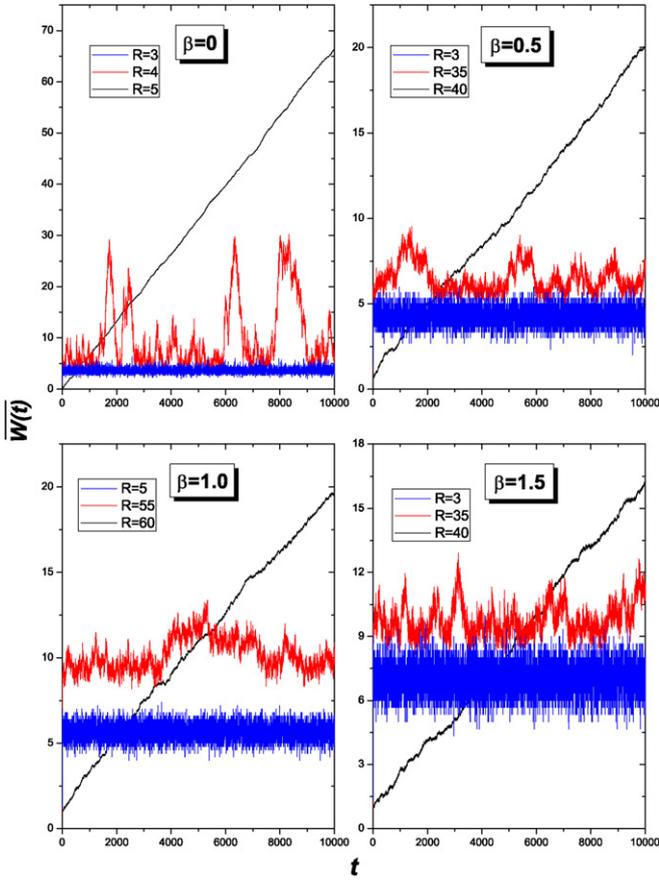
Fig. 3. The traffic rate process for free (red), critical (blue) and congested (black) states with different β. (For interpretation of the references to colour in this figure legend the reader is referred to the web version of this Letter.)

In Fig. 3, we report the average number of packets over all the nodes, $\overline{W}(t) = W(t)/N$, as a time series in free, critical and congested states, respectively. As time goes on, $\overline{W}(t)$ in the congested state will become too large to be plotted together with those in free and critical state. Therefore, each curve representing the congested state in Fig. 3 is anteriorly divided by a very large number. Those numbers are not the same for different simulations, and do not have any physical meanings, they are just used to make the figure more clear. The behaviors of $\overline{W}(t)$ in the free and congested states are very simple: In the former case, it fluctuates slightly around a very low value, while in the latter case, it increases linearly. However, the time series at the critical point is very complicated, we next detect its scaling property by using the detrended fluctuation analysis.

4.2. Detrended fluctuation analysis

The empirical study on the self-similar scaling behavior of traffic rate process can be firstly found in some pioneer Refs. [39,40]. The autocorrelation function and power spectrum are widely used to analyze the self-similar scaling behavior of data [41–44]. However, it is shown that all the above methods do not work very well for the non-stationary time series, and are less accurate than the detrended fluctuation analysis (DFA) [45–47], which has now been accepted as an important time se-

ries analysis approach and widely used especially for financial and biological data [48–52].

The DFA method is based on the idea that a correlated time series can be mapped to a self-similar process by an integration. Therefore, measuring the self-similar feature can indirectly tell us the information about the correlation properties [45,46]. Briefly, the description of the DFA algorithm involves the following steps.

(1) Consider a time series x_i, $i = 1, \ldots, N$, where N is the length of this series. Determine the *profile*

$$y(i) = \sum_{k=1}^{i} [x_k - \langle x \rangle], \quad i = 1, \ldots, N, \tag{3}$$

where

$$\langle x \rangle = \frac{1}{N} \sum_{i=1}^{N} x_i. \tag{4}$$

(2) Divide profile $y(i)$ into non-overlapping boxes of equal size t.

(3) Calculate the local trend $y_{\texttt{fit}}$ in each box of size t by a least-square fit of the series, and the detrended fluctuation function is given as

$$Y(k) = y(k) - y_{\texttt{fit}}(k). \tag{5}$$

(4) For a given box size t, we calculate the root-mean-square fluctuation

$$F(t) = \sqrt{\frac{1}{N} \sum_{1}^{N} [Y(k)]^2}, \tag{6}$$

and repeat the above computation for different box sizes t (i.e., different scales) to provide a relationship between F and t. If the curve $F(t)$ in a log-log plot displays a straight line, then $F(t)$ obeys the power-law form t^{H^u} with H^u the slope of this line.

As shown in Fig. 4, the mimic traffic rate process also exhibits the self-similar scaling behaviors at the critical point of phase transition. The scaling exponents calculated with DFA for different β are approximate $H^u \approx 1$, and the value of β has almost no effect on H^u. This value of H^u implies the $1/f$-type scaling in the power spectrum and the long-range correlated behavior in a wide range of scales. A very recent empirical study on the traffic rate process of a University Ethernet has demonstrated that the real Ethernet traffic displays a self-similarity behavior with scaling exponent approximated to 0.98 [53], which agrees well with the present result $H^u \approx 1$.

In order to confirm that the critical behavior only exists in the critical state (as is shown in some previous empirical studies) [29–32], we analysis the traffic rate processes in the free-flow state. Fig. 5 suggests that traffic rate processes in free state are completely different from those in the critical state, which exhibit a very weak long-range correlated behavior with exponent $H^u = 0.55$. Clearly, if the transporting time of each packet is exactly the same as the path length (i.e., a packet does not need to wait at any nodes in the path from its source to destination),
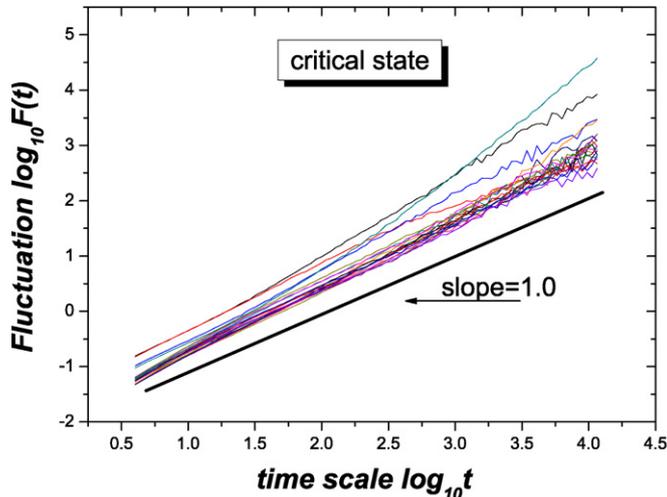
Fig. 4. The detrended fluctuation analysis of the traffic rate processes generated by the present model. All the data are obtained from the critical state, and the different curves represent the cases of different β from 0 to 1.9 at step 0.1. (For interpretation of the references to colour in this figure legend the reader is referred to the web version of this Letter.)
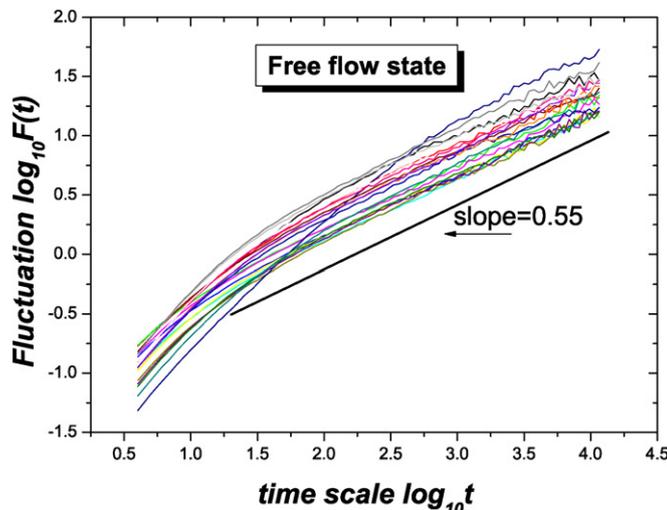

Fig. 5. The detrended fluctuation analysis of the traffic rate processes generated by the present model. All the data are obtained from the free-flow state, and the different curves represent the cases of different β from 0 to 1.9 at step 0.1. (For interpretation of the references to colour in this figure legend the reader is referred to the web version of this Letter.)

there should be no long-range correlation and the Hurst exponent will be $H^u = 0.5$. In the present model, even in the free-flow state, a packet may wait some time steps at some nodes before reaching to its destination. That is the reason leading to the weak long-range correlation. However, the waiting effect in the free-flow state is trifling for the Hurst exponent is close to 0.5.

5. Conclusion

In conclusion, we have proposed a traffic model based on packet routing strategy aiming at efficient paths instead of the shortest paths. This work may be useful for designing com-

munication protocols for highly heterogeneous networks since the present strategy can sharply enhance the throughput without any increase in its algorithmic complexity. The traffic rate process generated by this model exhibits critical self-similar scaling behavior with exponent approximated to 1, which implies the $1/f$-type scaling in the power spectrum and the long-range correlated behavior in a wide range of scales. The scaling behaviors of the present model is in good accordance with the empirical data, thus this model may have got hold of some key ingredients of the underlying mechanism of real traffic.

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