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An asymmetric full velocity difference car-following model

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Abstract

This paper presents a car-following model that considers the asymmetric characteristic of the velocity differences of the vehicles in a traffic stream. The problems of the prevalent general force (GF) model and the full velocity difference (FVD) model were solved. Furthermore, the optimal velocity (OV) model, the GF model, and the FVD model are proved to be the special cases of the proposed asymmetric full velocity difference (AFVD) model. The mathematical model is presented, followed by simulation analysis which demonstrates the properties of the AFVD model.
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Keywords: Car-following traffic model; Asymmetric full velocity difference model

1. Literature review and introduction

Car-following models describe the behavior of individual drivers in a stream of interacting vehicles traveling without making lane changes. Mathematical models on car-following have been proposed by many researchers. Early contributions include the linear car-following models by Chandler et al. [1] and Herman et al. [2]; the nonlinear approach by Reuschel [3], Pipes [4], Gazis et al. [5], and Newell [6]. Remarkable recent improvements include but are not limited to the works of Bando et al. [7], Helbing and Tilch [8], Treiber et al. [9], and Tomer et al. [10].

In 1995, Bando et al. [7] proposed an optimized velocity approach (OV) in which drivers are assumed to have the desire to maintain their optimal velocity in the traffic stream. The OV model takes the following form:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa[V(\Delta x_n(t)) - v_n(t)] \tag{1}$$

where V is defined as the optimal velocity a driver prefers based on the spacing to the leading vehicle and κ is the coefficient of sensitivity.

The OV model has attracted much attention because of its distinctive feature in representing real traffic flow characteristics such as the stop-and-go traffic and the evolution of traffic congestion. In 1998, Helbing and Tilch [8]

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conducted an experimental study with respect to the OV model and found that the OV model may result in impractical high acceleration and unrealistic deceleration. They further developed a generalized force model (GF) to show that when the velocity of the front vehicle is lower than that of the following vehicle, a new term representing the impact of the negative difference in velocity should be added to the OV model. Their model reads:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa[V(\Delta x_n(t)) - v_n(t)] + \lambda\Theta(-\Delta v_n(t))\Delta v_n(t) \tag{2}$$

where Θ is the Heaviside function and λ is another sensitivity constant. In the GF model, the dynamics of the vehicle α is presented by:

$$\frac{\mathrm{d}v_\alpha}{\mathrm{d}t} = \frac{v_\alpha^0 - v_\alpha}{\tau_\alpha} + f_{\alpha,\alpha-1}(x_\alpha, v_\alpha; x_{\alpha-1}, v_{\alpha-1}) \tag{3}$$

where v_α is the vehicle speed, v_α^0 is the desired optimal velocity and $f_{\alpha,\alpha-1}$ is the force with the following form:

$$f_{\alpha,\alpha-1} = \frac{V(s_\alpha) - v_\alpha^0}{\tau_\alpha} - \frac{\Delta v_\alpha \Theta(\Delta v_\alpha)}{\tau'_\alpha} \mathrm{e}^{-[s_\alpha - s(v_\alpha)]/R'_\alpha} \tag{4}$$

τ_α is the acceleration time and $\tau_\alpha = 1/\kappa$. τ'_α is the braking time, which should be smaller than τ_α. Δv_α is the velocity difference. s_α is the distance between two cars. R'_α can be interpreted as range of the braking interaction.

$$s(v_\alpha) = d_\alpha + T_\alpha v_\alpha. \tag{5}$$

The equation above is called velocity-dependent safe distance, where d_α is the minimal vehicle distance, and T_α is the safe time headway.

Considering that the GF model only considers the case where the velocity of the following vehicle is larger than that of the leading vehicle, Jiang et al. [11] pointed out that when the preceding car is much faster, the following vehicle may not brake even though the spacing is smaller than the safe distance. They proposed a full velocity difference model (FVD) that takes both positive and negative velocity difference into account. Their model reads:

$$\begin{aligned} &\frac{\mathrm{d}v_{n+1}}{\mathrm{d}t}(t) = \kappa[V(s_{n+1}) - v_{n+1}(t)] + \lambda\Delta v \\ &\Delta v = v_n(t) - v_{n+1}(t). \end{aligned} \tag{6}$$

If we simplify the GF model from Eqs. (3) and (4), we can find that

$$\frac{\mathrm{d}v_\alpha}{\mathrm{d}t} = \frac{V_\alpha(s_\alpha, v_\alpha) - v_\alpha(t)}{\tau_\alpha} - \frac{\Delta v_\alpha}{\tau''_\alpha}\Theta(\Delta v_\alpha). \tag{7}$$

It is clear that the FVD model uses λ to represent $\frac{1}{\tau''_\alpha}$. In the GF model, it is given by

$$\frac{1}{\tau''_\alpha} = \frac{1}{\tau'_\alpha \exp\{[s_\alpha - s(v_\alpha)]/R'_\alpha\}}$$

instead of a constant. Moreover, in the GF model, the term representing the velocity difference, i.e., $-\frac{\Delta v_\alpha}{\tau''_\alpha}\Theta(\Delta v_\alpha)$ is effective only when the spacing between the vehicle pair is sufficiently small compared to the safe distance $s(v_\alpha) = d_\alpha + T_\alpha v_\alpha$. Therefore, we can justify that the velocity difference part of the FVD model should be effective under the same situation.

2. The asymmetric full velocity difference model

A major deficiency of the FVD model lies in the fact that it models the velocity differences of vehicles symmetrically, which is not realistic. For example, the FVD model would inevitably lead to a problem that the following vehicle may not brake even if the distance to the leading vehicle is extremely close. This rarely happens in the real world because drivers would always like to keep a desired distance for safety. It is a common belief that vehicles' capability in deceleration is higher than in acceleration. Taking this asymmetric characteristic into account would greatly enhance the realism of car-following models.

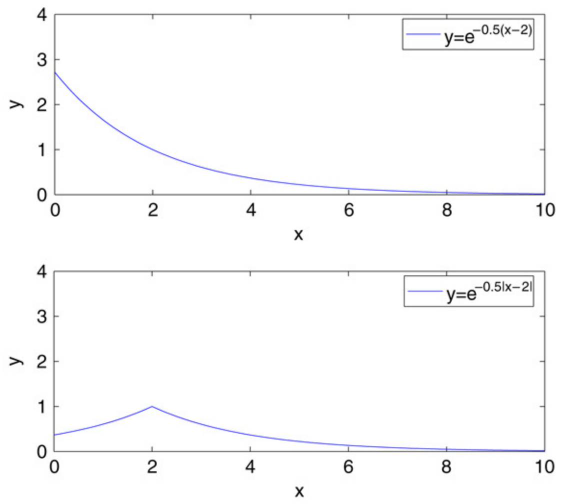


Fig. 1. Example exponential graph.

2.1. The effect of asymmetry and the new model

In [8], Helbing and Tilch mentioned the effect of the asymmetry on the breaking time τ'_α. It is intuitive that the counteracting term should decrease for large distance $s_\alpha \to \infty$, as well as when $s_\alpha \to 0$. Therefore, it is obvious that the λ term in the FVD model used for counteracting should be

$$\frac{1}{\tau'''_\alpha} \mathrm{e}^{-|s_\alpha - s(v_\alpha)|/R''_\alpha}.$$

The effect of asymmetry can be depicted by using the example exponential functions shown in Fig. 1. If we use the symmetric λ for both braking term and counteracting term, the algebraic sum of these two terms is almost zero. This means that there will be no braking activity even if the following vehicle is getting extremely close to the preceding vehicle. This modeled behavior obviously deviates from the real situation in which the following vehicle needs to brake sufficiently to avoid collision. On the other hand, if we use the $\frac{1}{\tau'''_\alpha} \mathrm{e}^{-|s_\alpha - s(v_\alpha)|/R''_\alpha}$ for the counteracting term, it will lead to an asymmetric form and the algebra sum will not be zero. As a result, the reasonable braking activity when the two vehicles are very close will not be eliminated.

The asymmetric characteristics were not considered in the GF model. Based on the above discussions, we give the following expression to consider the effects of asymmetric acceleration and deceleration in car-following:

$$\frac{\mathrm{d}v_n(t)}{\mathrm{d}t} = \kappa[V(\Delta x_n(t)) - v_n(t)] + \lambda_1 H(-\Delta v_n(t))\Delta v_n(t) + \lambda_2 H(\Delta v_n(t))\Delta v_n(t) \tag{8}$$

where H is the Heaviside function,

$$\lambda_1 = \frac{1}{\tau'_\alpha} \mathrm{e}^{-[s_\alpha - s(v_\alpha)]/R'_\alpha}$$

$$\lambda_2 = \frac{1}{\tau'''_\alpha} \mathrm{e}^{-|s_\alpha - s(v_\alpha)|/R''_\alpha}$$

τ'''_α and R''_α are two new parameters which need to be fixed by real data.

We can easily find the difference between the two sensitive coefficients. If we use the same parameter λ, then the λ_2 part will meet the problem that the vehicle will still accelerate when it is already very close to the front car. Since the λ_1 part is used for sufficient braking, it will increase sharply when two cars are getting closer. It is not logical to use the same parameter for the counteracting part. The λ_2 parameter was therefore proposed to represent the counteracting part. The most important point is that the counteracting rate will decrease when two cars are close enough.

2.2. Stability analysis

We present in this section the linear stability analysis of the proposed AFVD model. The stable solution of Eq. (8) is: $x_n^{(0)} = bn + ct, \quad b = L/N, \quad c = V(b)$, where b is the spatial distance between the vehicle pair and c is the constant velocity of the stable solution of the traffic flow. This solution corresponds to the stable traffic flow without jam.

Let us set

$$x_n = x_n^{(0)} + y_n, |y_n| \ll 1$$

and substitute the term into the above second order differential equation about $x_n(t)$ to get the equation about $y_n(t)$. Making the expansion of $y_n(t)$ at both sides of the equation, omitting the terms of higher order, we can get

$$\begin{cases} y_n'' = \kappa[f\Delta y_n - y_n'] + \lambda_1 H(-\Delta y_n')\Delta y_n' + \lambda_2 H(\Delta y_n')\Delta y_n' \\ f = V'(b). \end{cases} \tag{9}$$

The equation of y_n can be solved by the Fourier series expansion about $\mathrm{e}^{\mathrm{i}\alpha_k n}$, which is:

$$y_k(n, t) = \exp\{\mathrm{i}\alpha_k n + zt\}, \qquad \alpha_k = \frac{2\pi}{N}k \quad (k = 0, 1, 2, \ldots, N-1). \tag{10}$$

Substituting it into the equation of y_n, we can get the following equation about z:

$$z'' = \kappa[f(\mathrm{e}^{\mathrm{i}\alpha_k} - 1) - z] + \lambda z(\mathrm{e}^{\mathrm{i}\alpha_k} - 1). \tag{11}$$

Here,

$$\lambda = \begin{cases} \lambda_1, & \text{when } \Delta y_n < 0 \\ \lambda_2, & \text{while when } \Delta y_n > 0 \end{cases} \tag{12}$$

where $z = u + \mathrm{i}v$. In order to guarantee that u, v do exist and be the real numbers, the establishment must satisfy: $f = V'(b) < \frac{\kappa}{2} + \lambda$.

Specifically, the stable condition is:

$$f = V'(b) \begin{cases} < \dfrac{\kappa}{2} + \lambda_1 & \text{when } \Delta v_n < 0 \\ < \dfrac{\kappa}{2} + \lambda_2 & \text{when } \Delta v_n > 0. \end{cases} \tag{13}$$

Recall that in Jiang et al.'s FVD model, the stable condition is:

$$f = V'(b) < \frac{\kappa}{2} + \lambda \tag{14}$$

and in Bando's OV model, the stable condition is:

$$f = V'(b) < \frac{\kappa}{2}. \tag{15}$$

We may find that the result of the AFVD model is consistent with the full velocity difference model and optimal velocity model. In fact, the FVD model and OV model are two special cases of the proposed asymmetric full velocity difference model because if we let $\lambda_1 = \lambda_2 = \lambda$, the AFVD model represents the FVD model and the Eqs. (13) and (14) are identical; if we let $\lambda_1 = \lambda_2 = \lambda = 0$, both the AFVD model and the FVD model turns out to be the OV model and the three Eqs. (13)–(15) become identical.

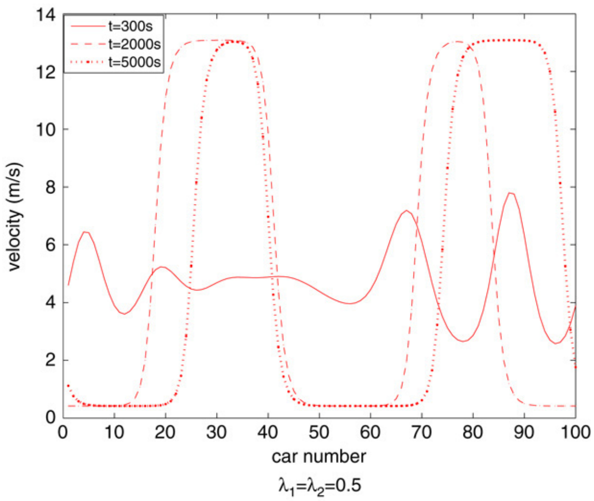


Fig. 2. Verification of the proposed simulation model: the stop-and-go chart of the FVD model.

3. Simulation

The purpose of the simulation study is to verify the proposed AFVD model and demonstrate its major properties. We use the following method in simulation:

$$\begin{cases} v_n(t+\Delta t) = v_n(t) + \Delta t \times \{\kappa[V(x_{n-1}(t) - x_n(t)) - v_n(t)] + \lambda_1[H(v_n(t) - v_{n-1}(t))][v_{n-1}(t) - v_n(t)] \\ \quad +\lambda_2[H(v_{n-1}(t) - v_n(t))][v_{n-1}(t) - v_n(t)]\} \\ x_n(t+\Delta t) = x_n(t) + \Delta t \times \dfrac{v_n(t) + v_n(t+\Delta t)}{2}. \end{cases} \tag{16}$$

To validate our simulation, let us first reproduce some results of the FVD model [11]. The time step of the simulation is 0.1 seconds and the distance between two vehicles is smaller than $\Delta x_c = 100$ m, length of the circle is $L = 1500$ m, and the number of vehicles $N = 100$. The initial perturbation is as follows:

$$\begin{cases} \begin{cases} x_n(t=0) = 1 \text{ (meter)} & \text{if } n = 1 \\ x_n(t=0) = (n-1)L/N & \text{if } n \neq 1 \end{cases} \\ v_n(t=0) = V(L/N). \end{cases} \tag{17}$$

Fig. 2 illustrates three velocity distributions obtained at the time steps of $t = 300$ s, 2000 s, 5000 s, from which we can find that the initially homogeneous traffic flow eventually developed into congested flow featured by the stop-and-go phenomenon in traffic stream.

Fig. 3 presents a reproduction of the hysteresis loops obtained from the FVD model. The Hysteresis loops are usually used to demonstrate the relationship between velocity and spacing after long enough simulation time (5000 s in this case).

The two simulation results were found to be identical with that of Jiang et al.'s FVD model, which confirmed the validity of our simulation model.

Next, we extend our simulation to verify the AFVD model. Assuming that the parameters τ'_α and τ'''_α, R'_α and R''_α are the same and we can find:

$$\lambda_1 = \frac{1}{\tau'_\alpha} e^{-[s_\alpha - s(v_\alpha)]/R'_\alpha} \geq \lambda_2 = \frac{1}{\tau'''_\alpha} e^{-[s_\alpha - s(v_\alpha)]/R''_\alpha}.$$

To make our simulation comparative to that of the FVD model, we set $\lambda_1 = 0.5$ and $\lambda_2 = 0.1, 0.2, 0.3, 0.4, 0.5$. The effect of the asymmetric acceleration and deceleration is demonstrated by the hysteresis loops and the stop-and-go chart. As can be seen from Fig. 4, the hysteresis loops obtained from the AFVD model are significantly different from that from the FVD model.

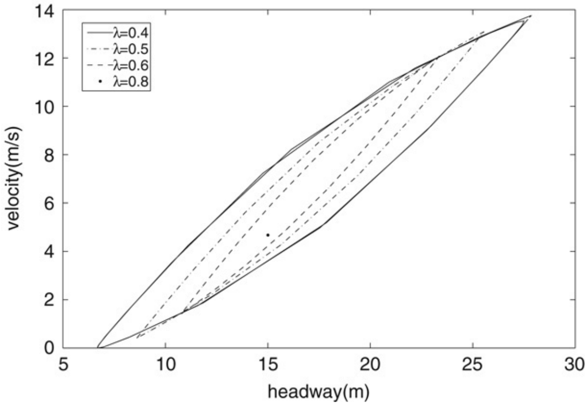


Fig. 3. Verification of the proposed simulation model: reproduction of the Hysteresis loops of the FVD model at different values of λ.

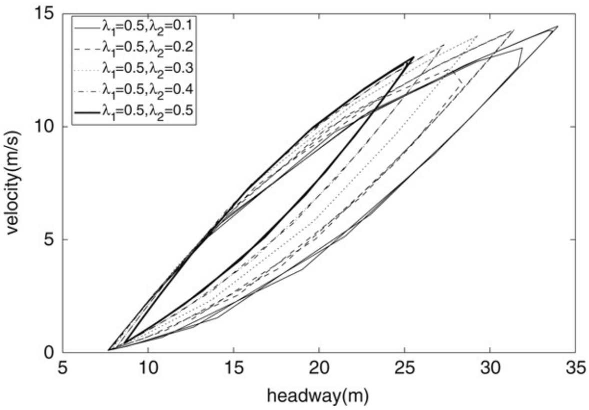


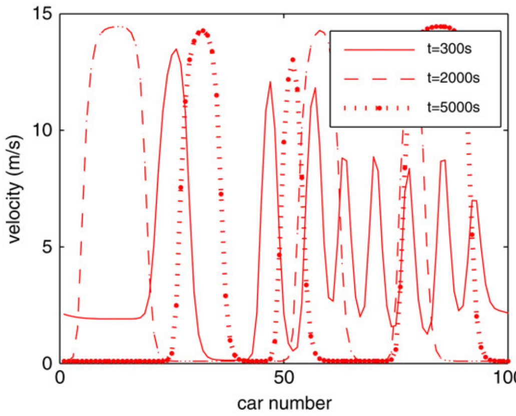
Fig. 4. Hysteresis loops of the AFVD model with fixed λ_1 and different λ_2.

We can find that if $\lambda_1 = \lambda_2 = 0.5$, the phase diagrams are identical to the result of the FVD model. However, we can also discover that when λ_2 becomes smaller, the loops have a clockwise rotation and become bigger in size, which indicates that the AFVD model's headways and velocities are bigger than that of the FVD model. Since the FVD model uses the symmetric velocity differences terms, the effect of the velocity difference parts should be weaker than the AFVD model. As a result, the fluctuation of the FVD model should be smaller than that of the AFVD model. This phenomenon is perfectly verified by the simulation.

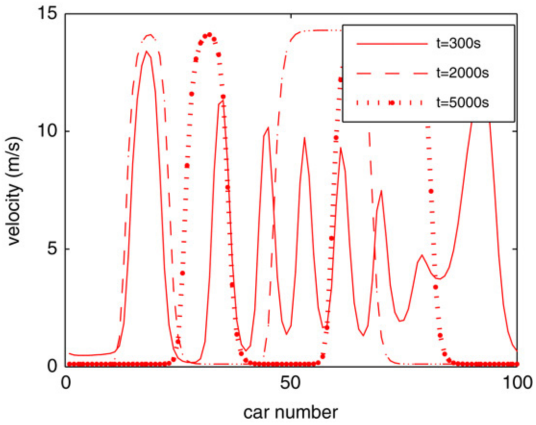
More differences can be found by further investigation of the stop-and-go chart. Fig. 5 depicts the stop-and-go traffic when $\lambda_1 = 0.5$ is fixed and λ_2 changes from 0.1 to 0.4.

Compared to the stop-and-go chart derived by the FVD model, it is clear that the FVD model is easier to get stable situation due to the almost same velocity difference form. On the other hand, we can find in the AFVD model that when the difference of λ_1 and λ_2 gets smaller, the function of the last two terms will counteract much more significantly. So it is not hard to understand why it is easier for the FVD model to get the stable condition than the AFVD model and the GF model. This tendency is demonstrated clearly by Fig. 5, which shows that the fluctuation increases accordingly when we decrease the value of λ_2.

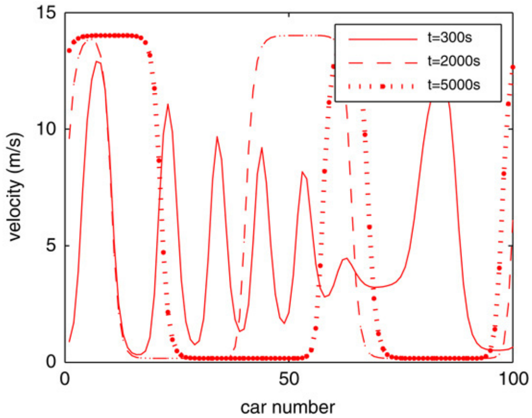
This characteristic can also be proved by the netto distance versus relative velocity diagram depicted in Fig. 6. The graph shows in a different form that the fluctuation of AFVD is larger than that of FVD. This is a simple confirmation that the easy achievement of stability in FVD is a result of the model's deficiency, i.e., the strong interaction between vehicle pairs are counteracted by the symmetric treatment of the sensitivity coefficient.



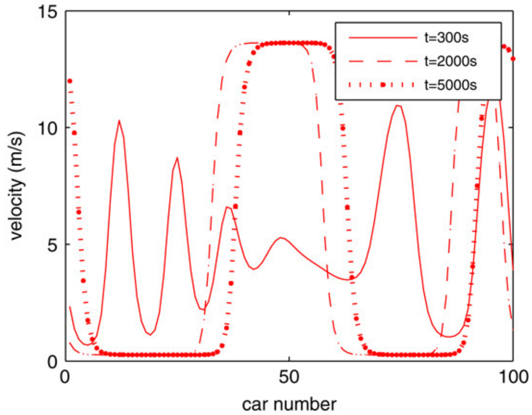
(a) $\lambda_1 = 0.5, \lambda_2 = 0.1$.



(b) $\lambda_1 = 0.5, \lambda_2 = 0.2$.



(c) $\lambda_1 = 0.5, \lambda_2 = 0.3$.



(d) $\lambda_1 = 0.5, \lambda_2 = 0.4$.

Fig. 5. The stop-and-go chart of the AFVD model with fixed λ_1 and different λ_2.

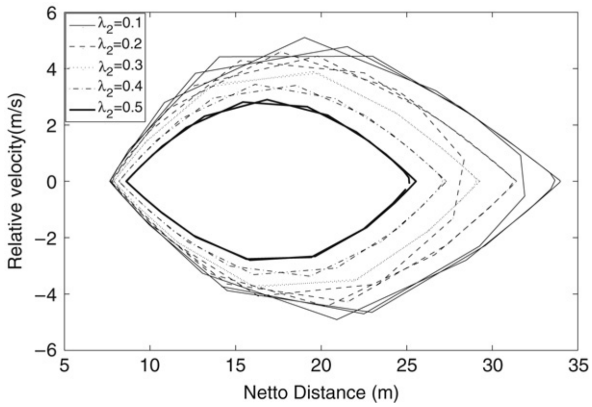


Fig. 6. Netto distance versus relative velocity with different λ_2.

4. Conclusion

The general force (GF) model and the full velocity difference (FVD) model are remarkable achievements among recent car-following models. The GF model has many distinctive features in modeling real traffic dynamics but does not consider the situation when the speed of the leading vehicle is much higher than that of the following vehicle, the follower may not brake even if the distance is smaller than the desired safe spacing. This was considered in the FVD model by the introduction of another velocity difference term to counteract the braking activity. However, we found that the FVD model over-counteracted the braking activity of the GF model and might lead to another unrealistic situation in which the following vehicle will not brake sufficiently even if the distance to the leading vehicle is extremely close. We further developed an asymmetric full velocity difference (AFVD) model to solve the problem. We used two sensitivity coefficients to model the velocity difference. It turned out that the OV model, the GF model, and the FVD model could be presented as the special cases of the AFVD model.

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