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An Efficient Control Strategy of Epidemic Spreading on Scale-Free Networks *

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We present a novel and effective method for controlling epidemic spreading on complex networks, especially on scale-free networks. The proposed strategy is performed by deleting edges according to their significances (the significance of an edge is defined as the product of the degrees of two nodes of this edge). In contrast to other methods, e.g., random immunization, proportional immunization, targeted immunization, acquaintance immunization and so on, which mainly focus on how to delete nodes to realize the control of epidemic spreading on complex networks, our method is more effective in realizing the control of epidemic spreading on complex networks, moreover, such a method can better retain the integrity of complex networks.

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Since the modelling of the small world network proposed by Watts and Strogatz,[1] and the scale-free network suggested by Barabási and Albert,[2] the studies of complex networks have attracted more and more interest in recent years. With the acquaintance of topological structures of complex networks, research of dynamical processes on complex networks have proved to be interesting and of vital importance. Examples of such dynamics include epidemic spreading,[3−16] the synchronization and pinning control of oscillators,[17,18] phase transitions[19] and so on. Among the above cases, epidemic spreading on complex networks is especially significant, since many disease outbreaks in biological systems can be viewed as the spread of an epidemic on complex networks.

When an epidemic spreads on a complex network, the topology structure of that network should be considered into the corresponding epidemic model. The susceptible/infective/susceptible (SIS) model on networks can be generally described as

$$\frac{dI_k(t)}{dt} = \lambda k(1 - I_k(t))\Theta(t, k) - I_k(t), \qquad (1)$$

where the infection rate $\lambda \in (0, 1]$ denotes the probability with which each susceptible node is infected if it is connected to one infected node. $\Theta(t, k)$ gives the probability that a randomly chosen link emanating from a node of connectivity k leads to infected nodes and has the following form

$$\Theta(t, k) = \sum_{k'} P(k'|k)I_{k'}(t), \qquad (2)$$

where the conditional probability $P(k'|k)$ means that a randomly chosen link emanating from a node of connectivity k leads to a node of connectivity k'.

For the spread of an epidemic on complex networks, there are two aspects which are mainly studied: One is the determination of the epidemic threshold λ_c, which is generally calculated by the ratio $\lambda_c = \langle k\rangle/\langle k^2\rangle$ with average degree $\langle k\rangle = \Sigma_k kP(k)$ and deviation degree $\langle k^2\rangle = \Sigma_k k^2 P(k)$, where $P(k)$ is the degree distribution of network. The disease will outbreak when the infection rate $\lambda > \lambda_c$, otherwise, the contagion of disease is self-limiting. Recently, a striking result is that the epidemic threshold λ_c is null if the epidemic spreads on a scale-free network with sufficiently large size ($\langle k^2\rangle \to \infty$ in this case).[3−5] The other one is the control strategy of an epidemic on complex networks, which has constructive meanings on the prevention of the epidemic spreading. Up to now, several famous control strategies have been proposed by researchers. For example, proportional immunization needs to immunize a very large fraction of nodes in a complex network. Targeted immunization, which we here also call node targeted control strategy to differentiate from our strategy, gives immunization to the highly connected individuals and has proven to be an effective control strategy when epidemics happen on scale-free networks. Acquaintance immunization calls for the immunization of random acquaintances of random individuals.[8] Although the mentioned strategies can prevent or restrain the spreading of an epidemic, they are prone to isolate the related individuals from the complex networks, which will destroy the integrity of the network (means the connectivity of the whole network). Generally, it is vi-

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tal importance to retain the integrity of a network. For example, in an Internet network, some hub routers can not be deleted directly from the Internet even though they are infected by a virus. The deletion of these hub nodes will break the integrity of the network, and the information can not be effectively spread on the Internet.

Next, we will propose an effective and novel control strategy: edge targeted control strategy, which deletes certain edges according to the edge's significance. As we will show later, the efficiency of our strategy is the same as or even better than the node targeted control strategy in the control of epidemic spreading on scale-free networks. Moreover, our strategy can better retain the integrity of a network.

The strategy is proposed as follows. The significance $C_{i,j}$ of an edge is calculated by the product of degrees of the two nodes i and j at both sides of this edge, i.e., $C_{i,j} = k_i k_j$, where k_i and k_j are the degrees of the node i and node j, respectively. After computing the significance of all edges, we rank the edges in descending order according to their significance. At each time step, we cut the edges with the highest rank. After deleting the most significant edges, we then compute the corresponding infection threshold $\lambda_c = \langle k \rangle / \langle k^2 \rangle$. By repeating the above process, the correlation between λ_c and the number of cut edges n can be obtained.

One thing which should be carefully noticed is that the above proposed strategy is not to give immunization to certain nodes in a network, but to delete certain edges to reduce the probability of an outbreak of an epidemic on complex networks. The operation process of our method is similar to the decoupling process method considered in Ref. [17], which is to enhance the synchronizability of a network. Then, the decoupling process just deletes one edge at each time step even though several edges have the same significance, and we will delete all edges if the edges have the same highest significance at each time step.

Just as we stated in the above context, for a scale-free network, the infection can prevail even though λ is very small because of the heterogeneity of the scale-free network, that is, some hub nodes are convenient foro the spread of an epidemic on a scale-free network. In view of this, our strategy is essentially to eliminate the effect of hub nodes in networks, such that the infection threshold λ_c can be increased dramatically. All simulations in this study are carried out on the scale-free Barabási–Albert network(BA) with size $N = 1000$, which can reflect many real complex systems. Starting from m_0 fully connected nodes, a new node with m $(m < m_0)$ edges is added to the existing different nodes at each time step. The newly added nodes connect to the old ones according to a preferential attachment mechanism in such a way that the probability $\prod_i$ of connecting to a existing node i is $\prod_i = k_i / \sum_j k_j$, where k_i is the degree of node i and the sum runs over all the existing nodes in the current network.[20]

In Fig. 1, we give a simulation to verify that our strategy (edge targeted control strategy) can better retain the integrity of the network. One can find that most of the nodes are still connected in the network even though 2000 edges (red) and 2500 edges (blue) are deleted according to their significances for the BA network with $N = 1000, m = 3, m_0 = 4$ (the total number of edges is 2994) and $N = 1000, m = 4, m_0 = 5$ (the total number of edges is 3890), respectively. This phenomenon can be explained as follows: according to our strategy, at each time step the edges linked to nodes with higher degrees will be removed from the network at first, but the node will not be isolated from the network since it is still linked by other edges.

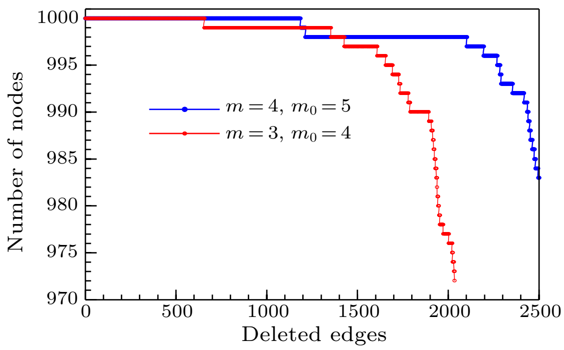

Fig. 1. The number of nodes (which is not separated from the network) with respect to the number of deleted edges according to their significance. For the BA network with $N = 1000$, $m = 3$, $m_0 = 4$, the number of connected nodes is more than 975 after 2000 edges are deleted according to their significance. More than 980 nodes remain in the network after 2500 edges are deleted according to their significance for the BA network with $N = 1000$, $m = 4$, $m_0 = 5$, respectively. All the data are obtained by averaging over 20 independent realizations.

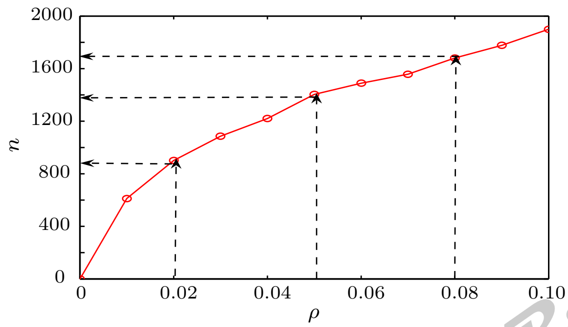

Fig. 2. The number n of deleted edges with respect to the fraction ρ of deleted nodes based on node targeted control strategy.

Though a few of nodes are removed from the scale-free network by the node targeted control strategy, the strategy will result in more deleted edges the degrees of hubs are very high. In order to compare the effect of node targeted control strategy with our proposed strategy, in Fig. 2 the relation between the number of deleted edges (n) and the fraction of deleted nodes (ρ) under the node targeted control strategy is demonstrated. As shown in Fig. 2, the number of deleted edges n is very large even though the fraction of deleted nodes ρ is small.

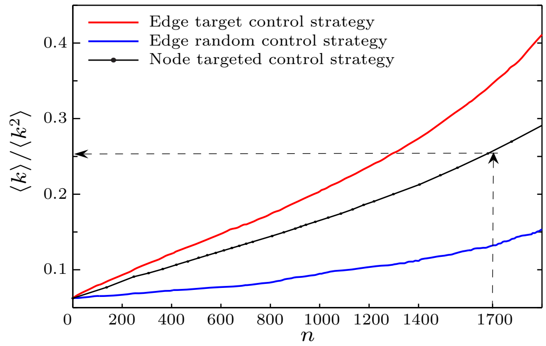

Fig. 3. Different changes of the ratio $\langle k\rangle/\langle k^2\rangle$ with respect to the number n of edges deleted based on edge targeted control strategy (red), node targeted control strategy (black) and edge random control strategy (blue), respectively.

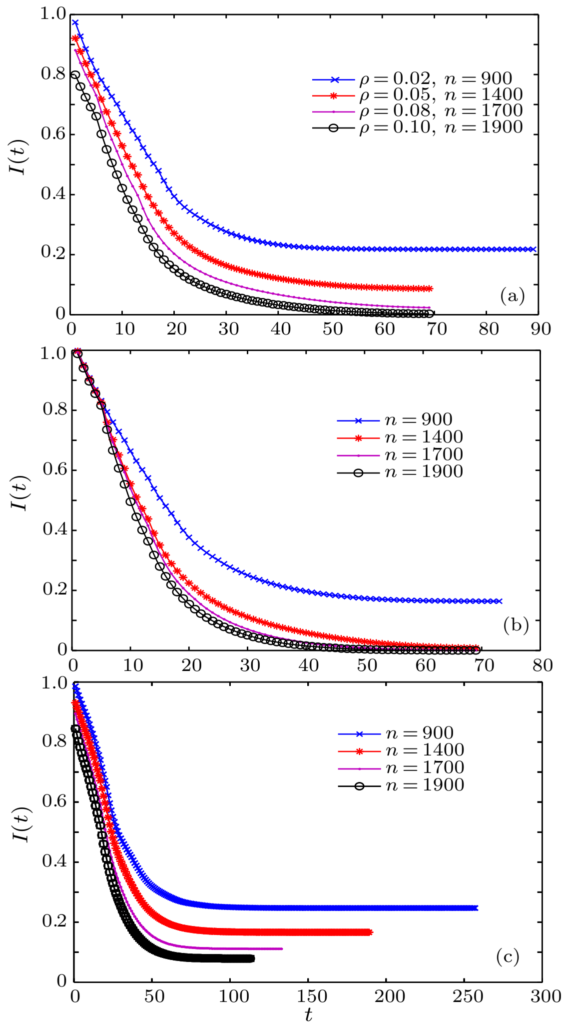

Fig. 4. Different evolvements of the density $I(t)$ of infected individuals with respect to the fraction ρ of nodes deleted under different control strategies: (a) for node targeted control strategy, (b) for edge targeted control strategy, (c) for edge random control strategy.

From Figs. 2–5, the simulations are carried out on the BA network with $N = 1000$, $m = 3$, $m_0 = 4$, and each data is obtained by averaging over 20 individual realizations.

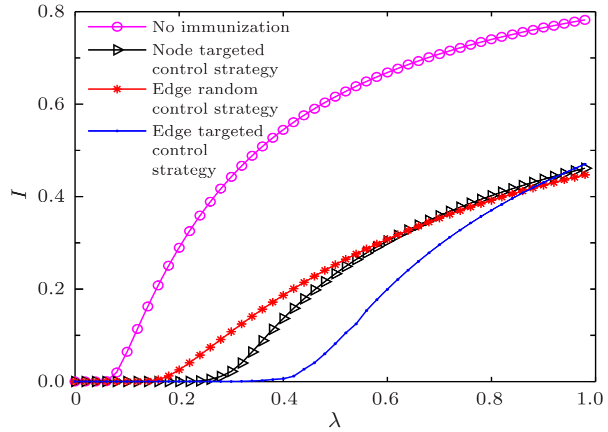

Fig. 5. Effects of different control strategies on epidemic threshold λ_c when 1900 edges are deleted (corresponding to $\rho = 0.1$ for the node targeted control strategy).

From Fig. 2, we can find that the number of deleted edges n is about 1900 when the fraction of deleted nodes becomes $\rho = 0.1$ under the node targeted control strategy. Thus, we here check the ratio $\langle k\rangle/\langle k^2\rangle$ vs the number of deleted edges, n, according to their significances in Fig. 3. After 1900 edges are deleted, the ratio $\langle k\rangle/\langle k^2\rangle$ is enhanced dramatically. In order to compare the effect of our strategy with the one which deletes edges randomly (here we name the edge random control strategy), we also study the change of ratio $\langle k\rangle/\langle k^2\rangle$ in Fig. 3 when the edges are deleted randomly. From this figure, we can see that the edge targeted control strategy is more effective than the edge random control strategy. Furthermore, one can also find that the ratio $\langle k\rangle/\langle k^2\rangle$ is about 0.41 when 1900 edges are deleted according to their significance, which is larger than the value $\langle k\rangle/\langle k^2\rangle \doteq 0.3$ under the node targeted control strategy with $n \doteq 1900$ (corresponding to $\rho = 0.1$).

We also use the SIS model on the scale-free network as an example to demonstrate the density of infected individuals $I(t)$ vs the number of deleted edges, n, for different strategies (including node targeted control strategy, edge targeted control strategy and edge random control strategy) with the same spreading rate $\lambda = 0.25$. In Fig. 4(a), we show the density $I(t)$ of infected individuals as a function of the fraction ρ of deleted nodes when node targeted control strategy is considered. Just as shown in Fig. 3, the ratio $\langle k\rangle/\langle k^2\rangle \le 0.25$ when $n \le 1700$, so the infected individuals are always prevalent when $n < 1700$. In addition, we obtain the ratio $\langle k\rangle/\langle k^2\rangle \doteq 0.3 > 0.25$ when $n = 1900$, as a result, the epidemic will be extinct (black curve). As shown in Fig. 3, one can find that the ratio $\langle k\rangle/\langle k^2\rangle > 0.25$ when the number of deleted edges satisfies $n \ge 1400$ based on our strategy. Then the epidemic will die out when $n \geq 1400$, which is also verified in Fig. 4(b). For the edge random control strategy, the ratio $\langle k\rangle/\langle k^2\rangle$ is always less than 0.25 even though 1900 edges are deleted randomly (see Fig. 3, blue curve). Therefore, the infected individuals will always prevail in the network, which is verified by Fig. 4(c).

Finally, we simulate the epidemic threshold λ_c under different control schemes as shown in Fig. 5. Obviously, our strategy can enhance the epidemic threshold λ_c dramatically, that is, the epidemic spreading on a complex network is suppressed successfully. Moreover, the strategy is better than the node targeted control strategy when $n = 1900$ edges are deleted. We also simulate other BA networks with different parameters, and find that the obtained results are all similar to above.

In summary, we have put forward a novel and efficient strategy for controlling the spread of an epidemic on scale-free networks: the edge targeted control strategy. By considering the ratio $\langle k\rangle/\langle k^2\rangle$ and the density $I(t)$ of infected individuals both as functions of the number of deleted edges according to our strategy, simulations have indicated that the proposed strategy can effectively inhibit epidemic spreading on complex networks, especially on scale-free networks. Moreover, our strategy can better retain the integrity of networks. Almost all previous control strategies only consider how to delete nodes. As a result, the integrity of the network may be badly destroyed, which may give unsatisfactory results in many practical applications. For example, the cascades failure phenomenon may emerge if some hub nodes are deleted from the networks.[21] Thus, our control scheme may be of great significance for preventing the spread of an epidemic in many real epidemic systems.

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