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Traffic Flow and Efficient Routing on Scale-Free Networks: A Survey

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Recently, motivated by the pioneering that have revealed the small-world effect and scale-free property of various real-life networks, many scientists are devoting themselves to studying complex networks. In this paper, we give a brief review of studies on traffic flow and efficient routing on scale-free networks, including traffic dynamics based on the global routing protocol, traffic dynamics based on the local routing protocol, and the critical phenomena and scaling behaviors of real and artificial traffic. Finally, perspectives and some interesting problems are discussed.

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I. INTRODUCTION

Many social, biological, and communication systems can be properly described as complex networks with vertices representing individuals or organizations and links mimicking the interactions among them [1–4]. One of the ultimate goals of the current studies on topological structures of networks is to understand and explain the workings of systems built upon those networks, for instance, to understand how the topology of the World Wide Web affects Web surfing and search engines, how the structure of social networks affects the spread of diseases, information, rumors, or other things, how the structure of a food web affects population dynamics, and so on. The increasing importance of large communication networks, such as the Internet [5], upon which our society survives, calls for high efficiency in handling and delivering information. Therefore, to understand the traffic dynamics and to find optimal strategies for traffic routing are ture one of the important issues we have to address. There have been many previous studies to understand and control traffic congestion on networks, with a basic assumption that the network had a homogeneous structure [6–10]. However, many real-life communication networks, such as the Internet [11] and the World-Wide-Web [12], display scale-free degree distribution [13,14]; thus, it would be great interest to study the traffic flow on scale-free networks. In this light, the traffic dynamics on complex networks has recently attracted a large amount of interest from the physical community.

In this paper, we will give a brief review of traffic dynamics on scale-free networks. This paper is organized as follow: In Section II and III, the traffic dynamics with global and local routing protocols are introduced, respectively. In Section IV, the critical phenomena and self-similarity scaling of real traffic and the artificial models are discussed. Finally, we sum up this paper in Section V.

II. TRAFFIC DYNAMICS BASED ON GLOBAL ROUTING PROTOCOL

In this section, we discuss the case where all topological information is available for each router. For simplicity, all the nodes are treated as both hosts and routers. The simplest model can be described as follows:

(1) At each time step, there are R packets generated in the system, with randomly chosen sources and destinations. Once a packet is created, it is placed at the end of the queue if this node already has several packets waiting to be delivered to their destinations.

(2) At each time step, each node, i, can deliver at most

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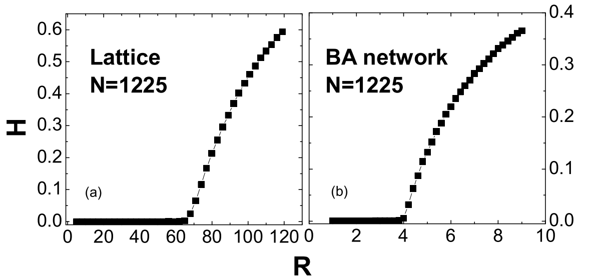

Fig. 1. Order parameter H versus R for (a) a two-dimensional lattice and (b) BA networks with the same size $N = 1225$ and average degree $\langle k \rangle = 4$. The delivering capability of each node is simply set as a constant $C = 1$. The shortest-path routing strategy yields $R_c^{\mathtt{Lattice}} \approx 60$ and $R_c^{\mathtt{BA}} \approx 4.0$.

C_i packets one step toward their destinations according to the routing strategy.

(3) A packet, once reaching its destination, is removed from the system.

We are most interested in the critical value R_c for which a phase transition takes place from free flow to congested traffic. This critical value can best reflect the maximum capability of a system for handling its traffic. In particular, for $R < R_c$, the numbers of created and delivered packets are balanced, leading to a steady free traffic flow. For $R > R_c$, traffic congestion occurs as the number of accumulated packets increases with time, simply become the capacities of nodes for delivering packets are limited. To characterize the phase transition, we use the following order parameter:

$$H(R) = \lim_{t\to\infty} \frac{C}{R}\frac{\langle \Delta W \rangle}{\Delta t}, \tag{1}$$

where $\Delta W = W(t + \Delta t) - W(t)$, with $\langle \cdots \rangle$ indicating an average over time windows of width Δt, and $W(t)$ is the total number of packets in the network at time t. Clearly, H equals zero in the free-flow state and becomes positive when R exceeds R_c.

Since in the Internet, the deviation of traffic from the shortest path is only about 10 % [15], one can assume that the routing process takes place according to the criterion of the shortest available path from a given source to its destination. Accordingly, firstly, we investigate the shortest-path routing strategy, which can be implemented in either of two ways, finding the shortest-path dynamically or following the fixed routing table. In the former case [16], for each newly generated packet, the router will find a shortest path between its source and destination; and then, the packet is forwarded along this path during the following time steps. In the latter case [17], for any pair of sources and destinations, one of all the shortest paths between them is randomly chosen, put into the fixed routing table, and is followed by all information packets. Compared with the dynamical routing algorithm and the information feed-back mechanism, the fixed routing algorithm is much more widely used in real communication systems for its obvious advantages in economical and technical costs [18,19]. Actually, the behaviors of those two cases are pretty much the same [16,17]; thus, we will not distinguish between them hereinafter.

If the delivering capability of each node is the same, the critical point R_c of a highly heterogeneous network will be much smaller than that of homogeneous network because when all packets follow their shortest paths, it will easily overload the heavily-linked router, and the congestion will immediately spread over all the nodes. Fig. 1 shows the order parameter H versus R for (a) a two-dimensional lattice with a periodic boundary condition and (b) the Barabási-Albert (BA) network [13,14] with average degree $\langle k \rangle = 4$. Clearly, the throughput, measured by the R_c of a regular network is much larger than that of scale-free networks.

To provide the theoretical estimate of the value of R_c, we first introduce the concept of betweenness centrality (see also the original concept of centrality [20, 21], the generalized concept of centrality [22], the physical meaning of betweenness centrality [23], and some recently proposed algorithms for calculating betweenness [24–26]). The betweenness centrality of a node v is defined as

$$B_v = \sum_{s\neq t} \frac{\sigma_{st}(v)}{\sigma_{st}}, \tag{2}$$

where σ_{st} is the number of shortest paths going from s to t and $\sigma_{st}(v)$ is the number of shortest paths going from s to t and passing through v. Below the critical value R_c, there is no accumulation at any node in the network, and the number of packets that arrive at node v is, on average, $RB_v/N(N-1)$. Therefore, a particular node will collapse when $RB_v/N(N-1) > C_v$. Considering the transferring capacity of each node is fixed to C and congestion occurs at the node with the largest betweenness centrality, R_c can be estimated as [16,27]

$$R_c = CN(N-1)/B_{\mathtt{max}}, \tag{3}$$

where $B_{\mathtt{max}}$ is the largest betweenness centrality of the network. This equation points out that a network of larger $B_{\mathtt{max}}$ has a smaller throughput.

For many real-life networks, the betweenness centrality is strongly correlated with degree. In general, the larger the degree, the larger the centrality. For many scale-free networks, it has been shown that the betweenness centrality approximately scales as $B(k) \sim k^{\mu}$ [28,29], where $B(k)$ denotes the average betweenness centrality over all the k-degree nodes. Therefore, in a heterogeneous network, there exist a few high-betweenness nodes, named *hub nodes*, which are easily congested. This is precisely the cause of low throughput of the scale-free networks.

To enhance the traffic capability, Zhao *et al.* proposed two traffic models [16], where the delivering capability of a node i is assigned as either $C_i = 1 + \beta k_i$ (Model I) or $C_i = 1 + \beta B_i$ (Model II). Here, $0 < \beta < 1$ is a control

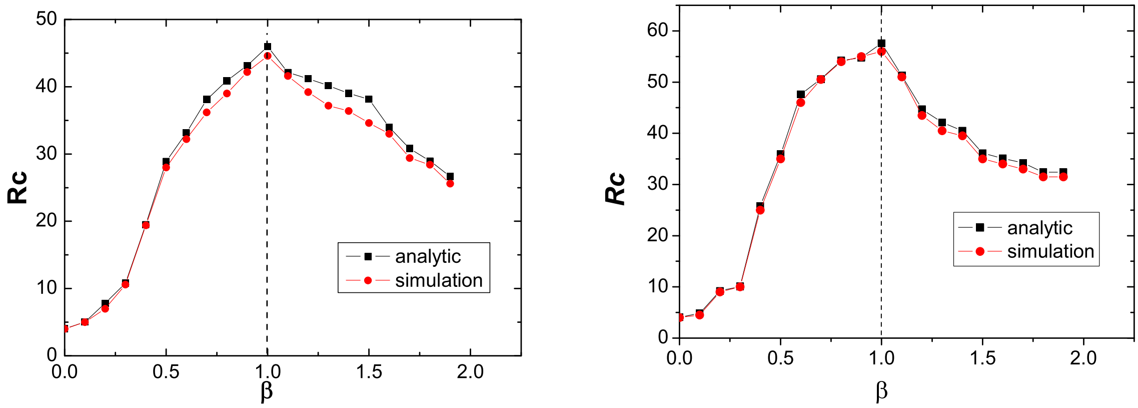

Fig. 2. (Color online) The critical R_c vs β for a BA network with an average degree $\langle k \rangle = 4$ and size $N = 1225$ (left panel) and $N = 1500$ (right panel). Both the simulation and the analysis demonstrate that the maximal R_c corresponds to $\beta \approx 1$. The results are averages over 10 independent runs.

parameter. As we have mentioned above (see Eq. (3)), it is clear that the throughput of the whole network will increase if the hub nodes have higher delivering capability though the total capability $\sum_i C_i$ remains unchanged. This work suggests a way to alleviate traffic congestion for highly heterogeneous networks: making nodes with large betweenness as powerful and efficient as possible for processing and transmitting information. Particularly, in model II, the throughput R_c is independent of the network topology.

However, two things hinder the application of those models. First, the capability/power distributions in some real networks are homogeneous, although their degree distributions are heterogeneous [30]. For example, in broadcasting networks, the forwarding capacity of each node is limited. Especially, in wireless multihop ad-hoc networks, each node usually has the same power, thus almost the same forwarding capacity [31]. Second, the structures of real networks evolve ceaselessly; thus, the degree and betweenness centrality of each node vary ever and again. By contrary, one cannot change the delivering capability of a node freely due to technical limitations.

For the case that each node has the same delivering capability C, Yan *et al.* proposed a novel routing algorithm to enhance the network throughput [32,33]. Note that, the path with shortest length is not necessarily the quickest way, considering the presence of possible traffic congestion and waiting time along the shortest path. Obviously, nodes with a larger degree are more likely to bear traffic congestion; thus, a packet will, an average, spends more waiting time to pass through a high-degree node. All too often, bypassing those high-degree nodes, a packet may reach its destination quicker than by taking the shortest path. For any path between nodes i and j as $P(i \to j) := i \equiv x_0, x_1, \cdots x_{n-1}, x_n \equiv j$, denote

$$L(P(i \to j) : \beta) = \sum_{i=0}^{n-1} k(x_i)^\beta, \tag{4}$$

where β is a tunable parameter. An *efficient path* between i and j is corresponds to the route that makes the sum $L(P(i \to j) : \beta)$ minimum. Obviously, $L_{min}(\beta = 0)$ recovers the traditionally shortest path length. All the information packets follow efficient paths instead of shortest paths.

In Fig. 2, we report the simulation results for the critical value R_c as a function of β on BA networks with sizes $N = 1225$ and $N = 1500$, which demonstrate that the optimal router strategy corresponding to $\beta = 1$ and the size of BA network do not affect the value of optimal β. In comparison with the traditional routing strategy (*i.e.* $\beta = 0$), the throughput R_c of the whole network is greatly improved by more then 10 times without any increase in the algorithmic complexity. By extending the concept of betweenness centrality to efficient betweenness centrality, that is, by using efficient paths instead of shortest paths in the definition of betweenness centrality, we can obtain analytic results according to Little's law [16,27,32]. The analytical results are also shown in Fig. 2, and agree very well with the simulations. In previous studies, the betweenness centrality was always considered to be a static topological measure of networks under the latent assumption that all the information packets went along the shortest paths from source to destination. The work of Yan *et al.* shows that this quantity (efficient betweenness) is determined both by the routing algorithm and the network topology; thus, one should pay more attention to the design of routing strategies. For example, a more intelligent router that can detour at an obstacle performs much better than a traditional router that just waits at an obstacle [34], and a recent work demonstrates that dynamical information can be

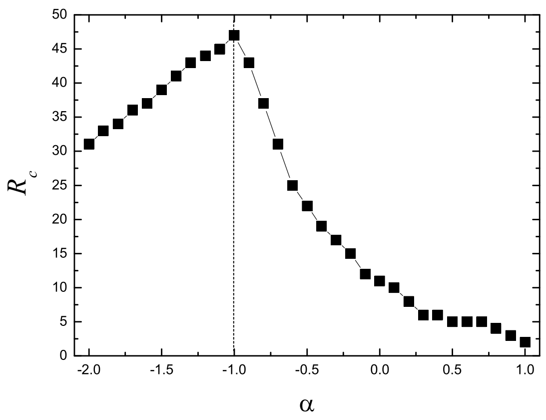

Fig. 3. Critical R_c vs α for a BA network with an average degree $\langle k \rangle = 10$ and size $N = 1000$. The delivering capability of each node is set as a constant $C = 10$. The results are averages over 10 independent runs.

used to design a more efficient routing strategy [35].

III. TRAFFIC DYNAMICS BASED ON LOCAL ROUTING PROTOCOL

Although a routing protocol using global topological information is very efficient, it is not practical for huge-size communication networks and evolving networks due to the technical limitation of the router because the router hardware is hard to design to have a capability to store much information or adapt dynamical information. Therefore, it is also very interesting and practical to study traffic behaviors on scale-free networks based on local information. The simplest network traffic model on a local protocol is the random-walk process, where each packet is delivered to a randomly selected neighbor as far as it reaches the destination. The random-walk on scale-free networks has been extensively explored previously [36–38]; however, it is far from real traffic became it can not reproduce self-similar scaling, as we will show in the next section.

Motivated by previous studies about searching engines [39, 40] and global routing strateg [32] on complex networks, Yin *et al.* proposed a traffic model using preferential selection among local neighbors [41]. In at model, to navigate packets, each node performs a local search among its neighbors. If a packet's destination is neighboring, it will be delivered directly to its target; otherwise, it will be forwarded to a neighbor j of its currently located node i according to the preferential probability

$$\Pi_{i\to j} = \frac{k_j^\alpha}{\sum_l k_l^\alpha}, \tag{5}$$

where the sum runs over the neighbors of node i, k_i is the degree of node i, and α is an adjustable parameter. Similar to the models mentioned in the last section, the first-in-first-out (FIFO) discipline is applied at the queue of each node. Another important rule, named path iteration avoidance (PIA), is that no edge can be visited more than twice by the same packet. With the delivering capability of each node set as a constant C, the simulation results show that the optimal performance of the whole system corresponds to $\alpha = 1$ (see Fig. 3). This optimal value can also be analytically obtained [42]. Furthermore, if the delivering capability of each node is proportional to its degree, the optimal value of α will shift to $\alpha = 0$ [42].

It is worthwhile to emphasize that the behavior of Yin *et al.*'s model [41] is similar to that of Yan *et al.*'s model [32], and the behavior of Wang *et al.*'s model [42] is similar to that of Zhao *et al.*'s model [16]. These resemblances indicate the existence of some common policies between the design of global and local routing protocols; that is, by passing the hub nodes or improving the delivering capability of these nodes can sharply enhance the throughput of the whole network.

Note that, each router in the present models [41, 42] needs to know all its neighbors' degrees and that a packet has to remember the links its has visited, which requires much intelligence for the system. It may damage the advantage of the local routing strategy because to implementing the PIA rule will make this system even more complicated than the one using a fixed routing algorithm. Also, the throughput of networks is very low without the PIA rule due to the many unnecessary visits along the same links by the same packets.

Another factor that affects the performance of local routing strategy is the area of information a router can make use of. Based on an artificial, directed World-Wide-Web model (see some recently proposed theoretical models of the directed World-Wide-Web [43, 44]), Tadić and Rodgars investigated a local routing protocol with an finite buffer size for each router and found that the next-to-nearest routing algorithm can perform much better than the nearest routing algorithm [45]. In this model, each packet follows a random-walk unless its destination appears within the current router's search area, and the next-to-nearest routing algorithm means the router can directly deliver a packet to its destination if this destination is within two steps.

IV. THE CRITICAL PHENOMENA AND SCALING BEHAVIORS OF TRAFFIC

Recent empirical studies on communication networks have found pervasive evidence of some surprising scaling properties. One example of such discoveries is that the traffic rates of both a given link in the Internet and a local Ethernet exhibit self-similarity (or fractal-like) scaling. The multifractal scaling is also found over a small time scale [9, 46–49]. These empirical studies describe perti-

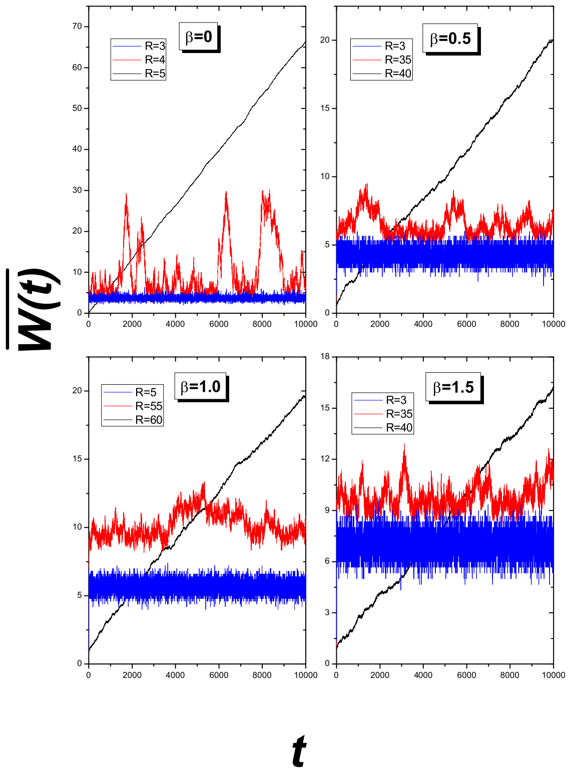

Fig. 4. (Color online) Traffic rate process for free (red), critical (blue), and congested (black) states with different β. All the data are obtained from an artificial traffic model [32].

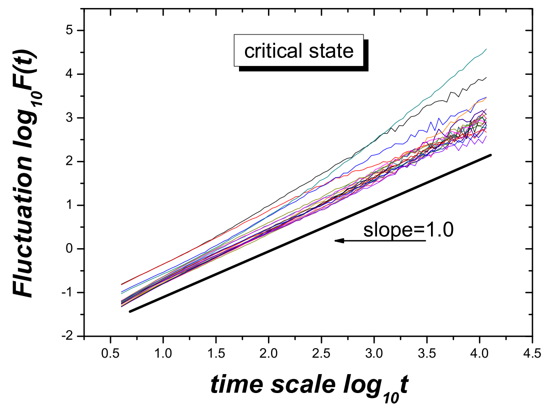

Fig. 5. (Color online) Detrended fluctuation analysis of the traffic rate processes generated by an artificial traffic model [32]. All the data are obtained from the critical state, and the different curves represent the cases of different β from 0 to 1.9 in steps of 0.1.

nent statistical characteristics of the temporal dynamics of the measured traffic rate process and provide ample evidence that these traces are consistent with the long-range correlated behavior. Furthermore, the existence of a phase transition between the free-flow phase and the congested phase in the Internet traffic was demonstrated by Takayasu *et al.* through both a round-trip time experiment [50,51] and a packet flow fluctuation analysis [52, 53]. They found that the real traffic exhibits a $1/f$-type scaling; however, this $1/f$ scaling can only be observed near the critical state [50–53].

Cai *et al.* [54] investigated the scaling behavior of a mimic traffic rate process near the critical point generated by the model of Yan *et al.* [32, 33]. Fig. 4 reports the average number of packets over all the nodes, $\overline{W}(t) = W(t)/N$, as a time series in free, critical, and congested states, respectively. The behaviors of $\overline{W}(t)$ in the free and the congested states are very simple: In the former case, it fluctuates slightly around a very low value while in the latter case, it increases linearly. However, the time series at the critical point is very complicated, and exhibits some large fluctuations like those have been observed in the real traffic [55]. The case of this phenomenon may be the use of a global routing strategy that leads to a possible long-range correlation, because this phenomenon cannot be detected in the random-walk model [36] and the model based on the local protocol [41, 42]. However, a very similar phenomenon is observed in a traffic model with a local protocol [56], where the buffer size of each router is finite.

In previous studies, the autocorrelation function and the power spectrum were widely used to analyze the self-similar scaling behaviors of both real [57, 58] and artificial [56] traffic data. However, it were shown that noneoys the above methods worked very well for the non-stationary effect [59], and all are less accurate than the detrended fluctuation analysis (DFA) proposed by Peng *et al.* [60, 61], which has now been accepted as an important time series analysis approach and is widely used, especially for financial and biological data [62–66].

The DFA method is based on the idea that a correlated time series can be mapped to a self-similar process by an integration. Therefore, measuring the self-similar feature can indirectly give us information about the correlation properties. Briefly, the description of the DFA algorithm involves the following steps:

(1) Consider a time series $x_i, i = 1, \cdots, N$, where N is the length of this series. Determine the *profile*

$$y(i) = \sum_{k=1}^{i}[x_k - \langle x\rangle], i = 1, \cdots, N, \tag{6}$$

where

$$\langle x\rangle = \frac{1}{N}\sum_{i=1}^{N} x_i. \tag{7}$$

(2) Divide the profile $y(i)$ into non-overlapping boxes of equal size t.

(3) Calculate the local trend $y_{\mathtt{fit}}$ in each box of size t by using a least-squares fit of the series, and the detrended fluctuation function is given as

$$Y(k) = y(k) - y_{\mathtt{fit}}(k). \tag{8}$$

(4) For a given box size t, we calculate the root-mean-square fluctuation

$$F(t) = \sqrt{\frac{1}{N}\sum_{1}^{N}[Y(k)]^2}, \tag{9}$$

and repeat the above computation for different box sizes t (*i.e.*, different scales) to provide a relationship between F and t. If the curve $F(t)$ in a log-log plot displays a straight line, then $F(t)$ obeys the power-law form t^H with H begin the slope of this line.

As Fig. 5, show the traffic rate time series generated by the model of Yan *et al.* [32] also exhibits self-similar scaling behaviors at the critical point of the phase transition. The scaling exponents calculated with the DFA for different β are approximately $H \approx 1$, and the value of β has almost no effect on H. This value of H implies a $1/f$-type scaling in the power spectrum and a long-range correlated behavior over a wide range of scales. A very recent empirical study on the traffic rate process of a University Ethernet has demonstrated that real Ethernet traffic displays a self-similarity behavior with a scaling exponent ≈ 0.98 [67], which agrees well with the present result $H \approx 1$.

V. CONCLUSION AND DISCUSSION

Studies of network traffic are now an the increase. Many new discoveries, especially the role of network heterogeneity, which can sharply reduce the traffic capacity, provide us with new scenarios for and problems in understanding and controlling traffic congestions. Physicists bring not only the new object named *scale-free networks* but also new methodologies much different from those usually used in engineering science. As an end of this brief review, we list a few interesting still unsolved problems.

Problem 1: The visual field of a router may be one of the most important factors that affects the traffic capacity of whole networks. In a random walk [36], the router knows nothing about the topological information; in the preferential routing strategy [41], the router knows the topological information of all its nearest neighbors; in the global routing protocol [16,32], each router knows all the whole topological information. Clearly, with wider visual field, the system can perform better. Tadić and Rodgers [45] proposed a local traffic model in which each router knows the topological information of all its nearest and next-nearest neighbors, which, as expected, has obviously higher throughput than the case where only the nearest-neighbors information is available. Up to now, there has been a lack of systemic study on the effect of a router's visual field on the traffic condition of a network, which may be worth some further work.

Problem 2: A router can memorize a huge amount of information about the shortest or the most efficient paths that, at least, can be used to implement the strategy of the fixed routing table. Even though, the current technology does not allour a router to do dynamical path-finding in large-size networks, so a problem related to the preceding one is what will happen if one mixes the global and the local protocols together; that is, a few routers in the networks can do global path-finding or have memorized the shortest/most efficient paths and others can only perform the local protocol. A further question is whether the addition of a few very powerful routers to a traffic system based on the local protocol can sharply enhance the network throughput and which locations these powerful routers should choose.

Problem 3: Although $\beta = 1$ corresponds to the optimal value of network throughput when using an efficient-path finding strategy [32], it is really a bad strategy when the traffic density is sparse because to bypass the hub nodes will increase the average distance between the source and the destination. If the traffic density is sparse, the strategy with $\beta > 0$ will waste time. Therefore, a natural question of how to use the dynamical information to improve the performance of network traffic is raised. Can we design some on-line algorithms to guide the information packets?

Problem 4: On one hand, in the network traffic dynamics, the maximal betweenness centrality $B_{\mathtt{max}}$ is the key factor that determines the network throughput R_c because the node having maximal betweeness centrality is the bottleneck of information traffic and, thus, is firstly congested. On the other hand, the node having maximal betweeness centrality is also the bottleneck that hinders the synchronization signal's communication; thus, a network with a higher $B_{\mathtt{max}}$ may have poorer synchronizability [68,69]. Therefore, we guess there may exist some common features between network traffic and network synchronization, although they seem completely unrelated. Actually, some recently proposed methods used to enhance the network synchronizability can also be used to enhance the network throughput [70–74]. An in-depth investigation would be great theoretical interest, and we want to know if those two different kinds of dynamics, traffic and synchronization, belong to some kind of "universality class."

Problem 5: The routing strategies for the real Internet [75] is of special interest for its practical significance. However, the real Internet is highly clustered and thus far has displayed a hierarchical structure [5], thus far from the extensively studied BA networks. Although there exists some highly-clustered models with hierarchical structures [76,77], they cannot capture the detailed topological properties of the real Internet. We have noticed that a recent model [78] aiming at the Internet is very close to reality; thus, it would be interesting to explore the difference between simulation results based on BA networks and the model of Zhou and Mondragón [78].

Problem 6: Previous studies have mainly focused on information flow and corresponding routing strategies. However, in the urban traffic, it is not the routes, but the drivers, that are intelligent. How can they make use of traffic information to shorten their travelling time [79], and well the selfish mean of each agent reduce system profit [80]?

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