Traffic dynamics based on local routing protocol on a scale-free network

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We propose a packet routing strategy with a tunable parameter α based on the local structural information of a scale-free network. As free traffic flow on the communication networks is key to their normal and efficient functioning, we focus on the network capacity that can be measured by the critical point of phase transition from free flow to congestion. Simulations show that the maximal capacity corresponds to $\alpha=-1$ in the case of identical nodes' delivering ability. To explain this, we investigate the number of packets of each node depending on its degree in the free flow state and observe the power law behavior. Other dynamic properties including average packets traveling time and traffic load are also studied. Inspiringly, our results indicate that some fundamental relationships exist between the dynamics of synchronization and traffic on the scale-free networks.

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I. INTRODUCTION

Complex networks can describe a wide range of systems in nature and society, therefore there has been a quickly growing interest in this area. Since the surprising small-world phenomenon discovered by Watts and Strogatz [1] and scale-free phenomenon with degree distribution following $P(k) \sim k^{-\gamma}$ by Barabási and Albert [2], the evolution mechanism of the structure and the dynamics on the networks have recently generated a lot of interest among physics community [3–8]. Due to the importance of large communication networks such as the Internet [9] and WWW [10] with scale-free properties in modern society, processes of dynamics taking place upon the underlying structure such as traffic congestion of information flow have drawn more and more attention from physical and engineering field [11–24]. In particular, interplay of traffic dynamics and evolution of networks structure also cannot be neglected for traffic system including Internet, WWW, and world-wide airport networks. Some previous works is mainly about the evolution of structure spurred by the increment of traffic [6–8] and others explore how different kinds of topologies affect the traffic dynamics taking place on them [25–29].

Processes of random walk on complex networks have been extensively investigated due to its wide application and fundamental dynamics which can be explained by theoretical analysis [30–32]. However, random walk is so simple that it could not reflect the real traffic system completely. Recent works proposed some models to mimic the traffic routing on complex networks by introducing packets (particles) generating rate as well as randomly selected source and destination of each packet [18,19,22,25,26]. These kinds of models also define the capacity of networks measured by critical generating rate. At this critical rate, a continuous phase transition from free flow state to congested state occurs. In the free state, the numbers of created and delivered packets are balanced, leading to a steady state. While in the jammed state, the number of accumulated packets increases with time due to the limited delivering capacity or finite queue length of each node. However, in these models, packets are forwarded either following the shortest path [14,16,19,25,26] or next-nearest-neighbor search strategy [18]. These routing rules allow each node to have the whole network's topological information or at least two order neighbor information, which may be practical for small or medium size networks but not for very large networks in modern society such as the Internet or WWW.

Another example of a process taking place on the network that has important practical applications is the network search or navigation on a network. Some previous works have focused on finding the optimal strategies for rapid target search based on local topological information (each node only knows the information of its neighbors) on the scale-free networks [33–37]. Such strategies with local information are favored in cases where there is a heavy communication cost to searching the network in real time. We hold that the studies of network search are also important for traffic systems because of the existence of packets routing from origin to destination and communication cost. However, few previous studies incorporate the search strategies and the traffic processes on networks. For this purpose, we model the traffic dynamics on scale-free networks with packets routing only based on local information to both minimize the packets delivering time and maximize the capacity of huge communication networks.

In this paper, we present a traffic model in which packets are routed only based on local topological information with a single tunable parameter α. In order to maximize the packets handling and delivering capacity of the networks which can be measured by an introduced order parameter η, the optimal α is sought out. We also investigate the dynamic properties in the steady state for different α including average number of packets versus node's degree, packets distribution and packets traveling time distribution. The dynamics right after the critical generating rate R_c exhibits some interesting prop-

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erties independent of α, which indicate that although the system enters the jammed state, it possesses partial capacity for forwarding packets. Our model can be considered as a preferential walk among neighbor nodes.

The paper is arranged as follows. In the following section we describe the model in detail, in Sec. III simulation and theoretical results of traffic dynamics are provided in both the steady and congested states, and in Sec. IV the work is concluded and compared with synchronization of coupled oscillators.

II. THE TRAFFIC MODEL

Recent studies indicate that many communication networks such as the Internet and WWW are not homogeneous similar to random and regular networks, but heterogeneous with degree distribution following the power-law distribution $P(k) \sim k^{-\gamma}$. Barabási and Albert proposed a simple famous model (BA for short) called scale-free networks [2], of which the degree distribution is in good accordance with real observations of communication networks. Here we use BA model with $m=5$ and network size $N=1000$ fixed for simulation. The traffic model is described as follows: at each time step, there are R packets generated in the system, with randomly chosen sources and destinations, and all nodes can deliver at most C packets towards their destinations. To navigate packets, each node performs a local search among its neighbors. If the packet's destination is found within the searched area, it is delivered directly to its target. Otherwise, it is forwarded to a nodes i, one of the neighbors of the searching node, according to the preferential probability:

$$\Pi_i = \frac{k_i^\alpha}{\sum_j k_j^\alpha}, \tag{1}$$

where the sum runs over the neighbors (searched area) of the searching node, k_i is the degree of node i and α is an adjustable parameter. Once a packet arrives at its destination, it will be removed from the system. The queue length of each node is assumed to be unlimited and the FIFO (first in first out) discipline is applied at each queue [18,19]. Another important rule called path iteration avoidance (PIA) is that a link between a pair of nodes cannot be visited more than twice by the same packet. Without this rule the capacity of the network is very low due to many times of unnecessary visiting along the same links by the same packets, which does not exist in the real traffic systems. For simplicity, we treat all nodes as both hosts and routers for generating and delivering packets. The node capacity C, that is the number of data packets a node can forward to other nodes each time step, is assumed to be either a constant for simplicity or $C=k$ corresponding to the degree of each node. In the first model of next section we set $C=10$, and $C=k$ in the second model for the last part.

III. SIMULATION AND THEORETICAL RESULTS

One of the most interesting properties of traffic system is the packets handling and delivering capacity of the whole network. As a remark, it is different between the capacity of the network and the nodes. The capacity of each node is set to be a constant C, while the capacity of the entire network is measured by the critical generating rate R_c at which a continuous phase transition will occur from free state to congestion. The free state refers to the balance between created and removed packets at the same time. If the system enters the jammed state, packets will continuously accumulate in the system and only a few packets can reach their destinations. In order to describe the critical point accurately, we use the order parameter introduced in Ref. [17]:

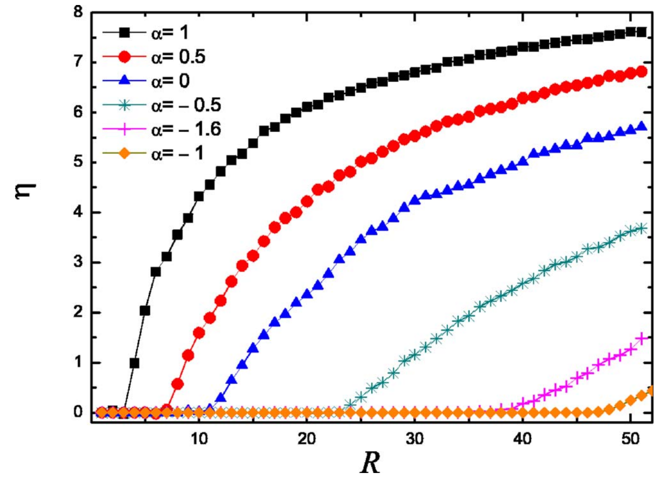

FIG. 1. (Color online) The order parameter η versus R for BA network with different free parameter α. Other parameters are networks size $N=1000$ and $C=10$.

$$\eta(R) = \lim_{t\to\infty} \frac{C}{R} \frac{\langle \Delta N_p \rangle}{\Delta t}, \tag{2}$$

where $\Delta N_p = N(t+\Delta t) - N(t)$ with $\langle \cdots \rangle$ indicates average over time windows of width Δt and $N_p(t)$ represents the number of data packets within the networks at time t. For $R < R_c$, $\langle \Delta N \rangle = 0$ and $\eta = 0$, indicating that the system is in the free state with no traffic congestion. Otherwise for $R > R_c$, $\eta \to r$, where r is a constant larger than zero, the system will collapse ultimately. Therefore a phase transition occurs at $R=R_c$ and R_c is the maximal generating rate under which the system can maintain its normal and efficient functioning. Thus the maximal handling and delivering capacity of the system is measured by R_c. As shown in Fig. 1, the order parameter η versus generating rate R with a different value of parameter α is reported. One can see that, for all different α, η is approximately zero when R is small; it suddenly increases when R is larger than the critical point R_c. It is easy to find that the capacity of the system is not the same for different α. Thus a natural question arises: what is the optimal value of α for maximizing the network's capacity? Simulation results demonstrate that the optimal R_c corresponds to $\alpha \approx -1$ as shown in Fig. 2. This result indicates that when $\alpha = -1$, the system's capacity can be enhanced maximally. Compared to previous work by Kim *et al.* [34], one of the best strategies is PRF corresponding to our strategy with $\alpha = 1$. By adopting this strategy a packet can reach

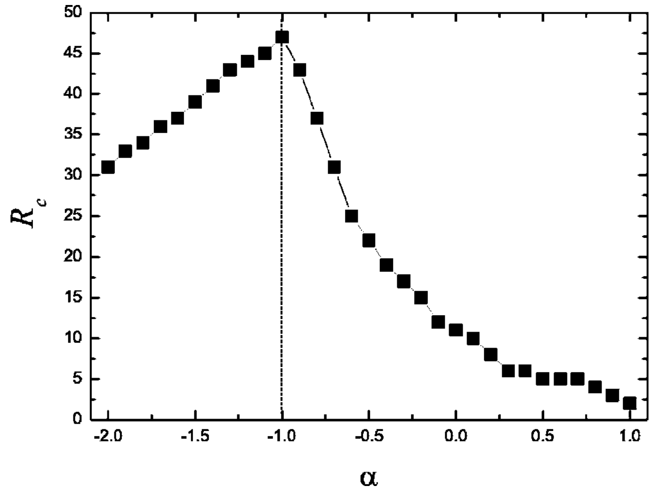

FIG. 2. The critical R_c versus α with network size $N=1000$ and constant node capacity $C=10$. The maximum of R_c corresponds to $\alpha=-1$ marked by a dotted line.

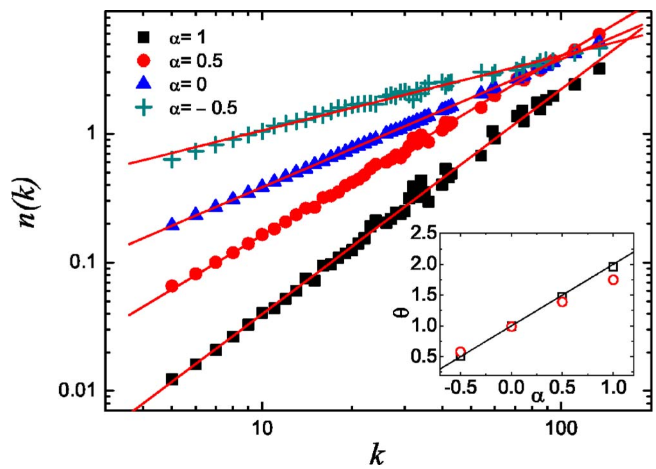

FIG. 3. (Color online) Average number of packets of nodes $n(k)$ depending on degree k with PIA for different α, the network size $N=1000$. In the inset, the line is the theoretical prediction, the red circles and dark squires are the values measured from the simulation with PIA for $N=1000$ and without PIA for $N=5000$, respectively.

its target node most rapidly without considering the capacity of the network. This result may be very useful for search engines such as Google, but for traffic systems the factor of traffic jam cannot be neglected. Actually, the average time the packets spend on the network can also be reflected by system capacity. It will indeed reduce the network's capacity if packets spend much time within the network. Therefore, choosing the optimal value of $\alpha=-1$ not only maximizes the capacity of the system but also minimizes the average delivering time of packets in our model.

To better understand why $\alpha=-1$ is the optimal choice, we also investigate the average number of packets $n(k)$ of a node as a function of its degree k. The average is over the nodes with the same degree. Assume that $n_i(k)$ is the number of packets of node i at time t and k_i is the degree of node i. Considering the contribution of both received and delivered packets of node i to the change of $n_i(t)$, the evolution of $n_i(t)$ in the free flow state can then be written as

$$\frac{dn_i(t)}{dt} = -n_i(t) + \sum_{j=1}^{N} A_{ij} n_j(t) \Pi_i = -n_i(t) + \sum_{j=1}^{N} A_{ij} n_j(t) \frac{k_i^\alpha}{\sum_{l=1}^{N} A_{jl} k_l^\alpha}, \tag{3}$$

where the sum runs over all the nodes of the network and A_{ij} is the element of the adjacency matrix. If there exist a link between i and j, $A_{ij}=1$. Otherwise, $A_{ij}=0$. The first term on the right side of Eq. (3) represents the delivered packets from i to its neighbors. In the free state, no congestion occurs at all nodes and each node can forward all the packets of it. Therefore, the first term is $-n_i(t)$. The second term represents the packets received from the neighbors of node i at time t. Take into account that the assortative mixing of BA network is zero [38], i.e., the average neighbors' degree of each node is the same, therefore we can get

$$\sum_{l=1}^{N} A_{jl} k_l^\alpha = \sum_{l=1}^{N} A_{jl} W = k_j W, \tag{4}$$

where W is a constant. Due to the balance between the number of created and delivered packets of each node in the free flow state $dn_i(t)/dt=0$. Then Eq. (3) can be simplified to

$$n_i = \sum_{j=1}^{N} A_{ij} n_j \frac{k_i^\alpha}{k_j W}. \tag{5}$$

Equation (5) is not easily solved so we expect a power-law scaling relationship as

$$n_i = C k_i^\theta, \tag{6}$$

where C is a constant. Substitute Eq. (6) into Eq. (5), we get

$$C k_i^\theta = \frac{C k_i^\alpha}{W} \sum_{j=1}^{N} A_{ij} k_j^{\theta-1}. \tag{7}$$

Combining Eqs. (4) and (7), we obtain

$$C k_i^\theta = \frac{C k_i^\alpha}{W} k_i W = C k_i^{1+\alpha}. \tag{8}$$

Then the relationship between $n(k)$ and k is given by

$$n(k) \sim k^\theta, \tag{9}$$

where $\theta=1+\alpha$.

In order to test Eq. (9), we simulate the $n(k)$ versus k for different α, as shown in Fig. 3. The power-law behavior is found as predicted from the analytic result. We compare the simulation results with theoretical result as shown in the inset of Fig. 3. One can find that with the PIA rule, the simulations are not well confirmed with the analytic predictions for large α. However, when we remove PIA and extend the network size to $N=5000$, we see that the theory matches the simulated results perfectly. Therefore, the inconsistency be-

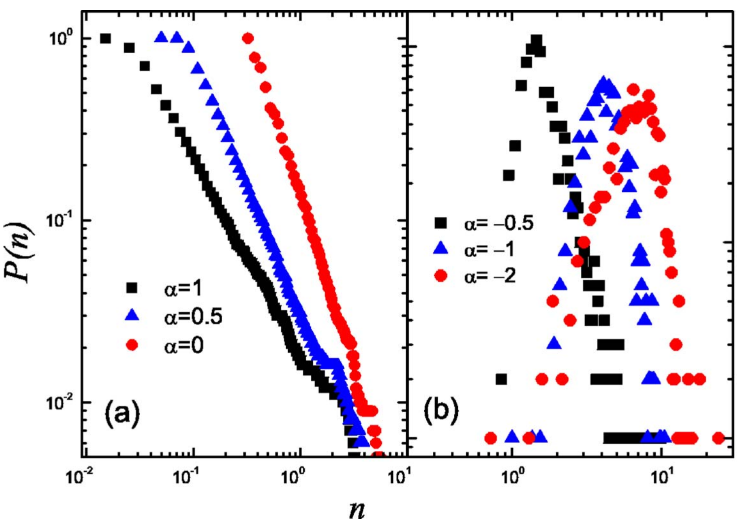

FIG. 4. (Color online) The distribution of packets on each node for different chosen parameter α with (a) $\alpha > 0$ and (b) $\alpha < 0$. In (a) $P(n)$ follows power-law distribution, while in (b) $P(n)$ approximately exhibits a Poisson distribution.

tween theoretical and simulated results is mainly due to the regularity of PIA and finite size effect.

We study the simulations of packets distribution $P(n)$ vs n for different α, where n represents the number of packets and $P(n)$ is the probability that a given node has n packets. As Fig. 4(a) shows, when $\alpha \geqslant 0$, the packets distribution follows a power law, which indicates the highly heterogenous traffic on each node. Combining Figs. 3 and 4, we can conclude that some nodes with large degree bear severe traffic congestion, while few packets pass through the others. The heterogenous behavior more obviously corresponds to the slope reduction with α increase from zero. Due to the same delivering capacity of all nodes, this phenomenon will undoubtedly do harm to the system because of the severe overburden of a few numbers of nodes. In contrast to Fig. 4(a), Fig. 4(b) shows that a better condition of the networks with queue length approximately displays the Poisson distribution, which represents the homogenous traffic on each node. From this aspect, we find that the capacity of the system with $\alpha < 0$ is larger than that with $\alpha > 0$. But it is not the whole story. In fact, the system's capacity is not only determined by the capacity of each node, but also by the actual traveling path length or traveling time of packets on the network from its source to destination. Suppose that if all packets bypass the large degree nodes, it will also cause the inefficient routing for ignoring the important effect of hub nodes on scale-free networks. Due to the competition of these two factors, there should exist an optimal parameter corresponding to the maximal network capacity. According to Eq. (9), $\alpha = -1$ leads to $\theta = 0$, which means after the transient time, the packets distribute averagely among all nodes. With respect to the identical packets delivering ability of nodes, this average effect results in the maximal network capacity.

As mentioned above, packets traveling time is also an important factor for enhancing the network's capacity. The traveling time τ of a packet is defined as the time spent by the packet in the system. Observations of real computer networks reveal that the packets traveling time has a power-law distribution. In Fig. 5 we report the distribution of packets traveling time for different α in the free flow state. The distribution is taken from packets which move within a total time window up to 50 000 time steps. In the steady state, almost no congestion on nodes occurs and the time of packets waiting in the queue is negligible, therefore, the traveling time is approximately equal to the actual path length of packets. In our model, it is observed that when α is larger than one, power-law behavior emerges, which is confirmed with empirical observation. when α is reduced, the distribution approaches to the exponential distribution. The power-law behavior reflects that most of packets can arrive at their destinations in a short time while small quantities of packets need to spend very long time to find their target nodes. The large α depicted in Eq. (1) represents that packets tend to move to the nodes with large degree (connectivity), which makes the hub nodes fully used and results in the power-law distribution of traveling time. This result indicates that in real computer networks hub nodes may also afford heavy traffic due to the empirical findings of packets traveling time.

Traffic dynamics in the jammed state is also interesting for alleviating traffic congestion. Figure 6 displays the evolution of $N_p(t)$, i.e., the number of packets within the network with distinct R. α is fixed to be -1.5 and R_c for α

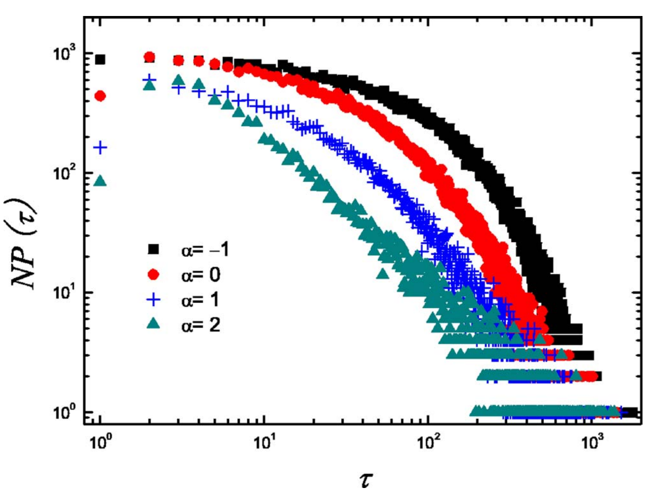

FIG. 5. (Color online) Packets traveling time distribution for different α all in the steady states. The simulation last for 50 000 time steps.

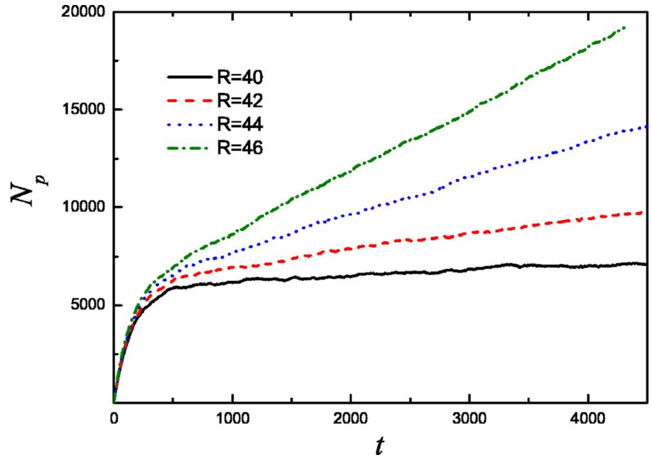

FIG. 6. (Color online) The evolution of N_p for $R>R_c$. Here, α_c takes -1.5 corresponding to the critical point $R_c=39$.

$=-1.5$ is 39. All the curves in this figure can be approximately separated into two ranges. The starting section shows the superposition of all curves, which can be explained by the fact that few packets reach their destinations in a short time so that the increasing velocity of N_p is equal to R. Then after transient time, N_p turns to be a linear function of t. Opposite to our intuition, the slope of each line is not $R-R_c$. We investigate the increasing speed of N_p depending on R by choosing different parameter α. In Fig. 7(a), in the congested state N_p increases linearly with the increment of R. Surprisingly, after x axis is rescaled to be $R-R_c$, three curves approximately collapse to a single line with the slope ≈ 0.7 as shown in Fig. 7(b). On one hand, this result indicates that in the jammed state together with R not being so large, the dynamics of the system do not depend on α. On the other hand, the slope less than 1 reveals that not all the $R-R_c$ packets are accumulated in the network per time step, but about 30% of the packets do not pass through any congested nodes, thus they can reach their destinations without contributing to the network congestion. This phenomenon also

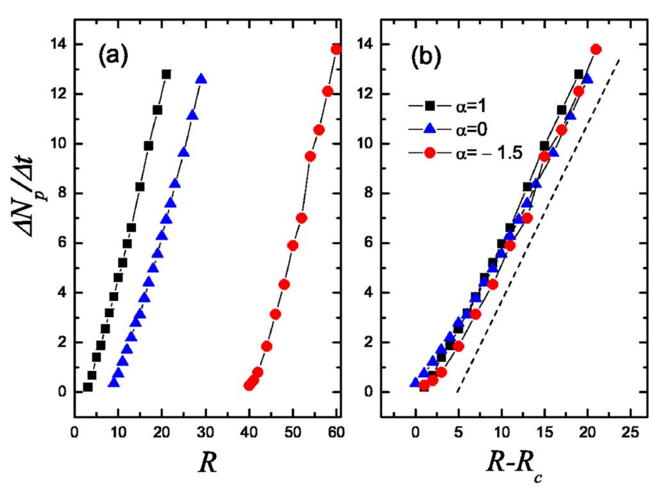

FIG. 7. (Color online) The ratio between ΔN_p and time step interval Δt versus R (a) and versus $R-R_c$ the rescaling of R (b) for different α. In (b) three curves collapse to a single line with the slope ≈ 0.7 marked by a dashed line.

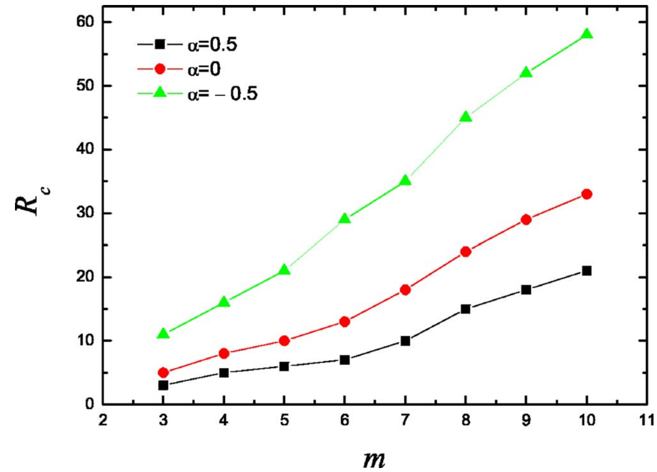

FIG. 8. (Color online) The variance of R_c with the increasing of m.

shows that when R is not too large in the congested state, the congested nodes in the network only take the minority, while most other nodes can still work. Therefore, the congestion of the system can be alleviated just by enhancing the delivering ability of a few number of heavily congested nodes. Furthermore, we study how the critical point R_c is affected by the link density of the BA network. As shown in Fig. 8, increasing m obviously enhances the capacity of the BA network measured by R_c due to the fact that with high link density, packets can more easily find their target nodes. Suppose that if the network is a fully connected graph, thus all packets can reach their destinations in one time step and the capacity of the system is the product of node's capacity C and network size N.

Considering the existence of different handling and delivering ability of nodes, we propose that the second model with C is not a constant but proportional to the degree of each node, i.e., $C_i=k_i$. From the aspect of self-organization, if a router is very important and usually bears heavy traffic, its delivering ability may be updated to adapt traffic congestion. Thus, during the evolution of the communication network, the delivering ability of each node may not be the same and busier traffic often passes through larger degree nodes. For simplicity, we assume C of a node is proportional to its degree k. The main difference between these two models is the optimal value of parameter α. Compared to the first model with $C=$ const, the optimal parameter value changes to $\alpha=0$, as shown in Fig. 9. This observation demonstrates that under the conditions of heterogenous forwarding capacity of each node, the random walk routing strategy is the best choice.

IV. CONCLUSION AND DISCUSSION

Motivated by the problem of traffic congestion in large communication networks, we have introduced a routing strategy based only on local information. Influenced by two factors of each node's capacity and navigation efficiency of packets, the optimal parameter $\alpha=-1$ is obtained maximizing the whole system's capacity. Dynamic behavior such as

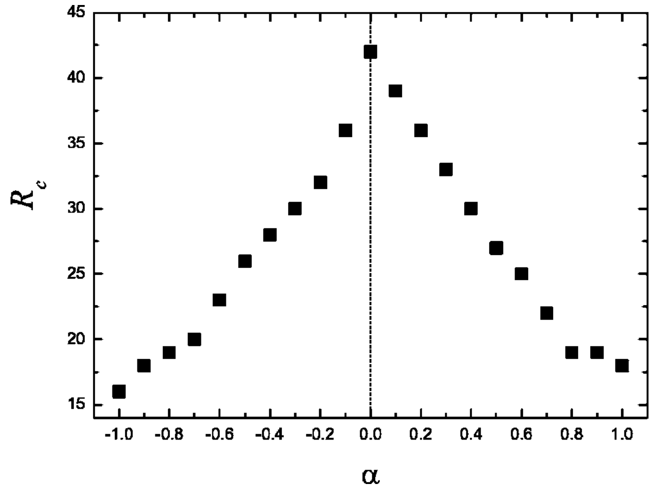

FIG. 9. The critical generating rate R_c versus α in the case of node capacity proportional to its degree $C_i = k_i$. Here, $k_{\min}$ is set to be 3 and network size $N=1000$. The maximum of R_c corresponds to $\alpha=0$ marked by a dotted line.

increasing velocity of N_p in the jammed state shows the universal properties which do not depend on α. In addition, the property that scale-free network with occurrence of congestion still possesses partial delivering ability suggests that only improving processing ability of the minority of heavily congested nodes can obviously enhance the capacity of the system. The variance of critical value R_c with the increment of m is also discussed. Our study may be useful for designing communication protocols for large scale-free communication networks due to the local information the strategy only based on and the simplicity for application. The results of the current work may also shed some light on alleviating the congestion of modern technological networks.

Recent work on network synchronization by Motter, Zhou, and Kurths [39] suggests that perhaps there are some fundamental relationships between traffic dynamics and synchronization of coupled oscillators on complex networks. In their work, a concept of coupling strength is proposed as a function of degree of nodes

$$G_{ij} = \frac{L_{ij}}{k_i^{\beta}}, \tag{10}$$

where β is a tunable parameter and they consider complete synchronization of linearly coupled identical oscillators

$$\frac{dx_i}{dt} = f(x_i) - \sigma \sum_{j=1}^{N} G_{ij} h(x_j), \quad i = 1,2, \ldots ,N, \tag{11}$$

where $L=(L_i j)$ is the usual (symmetric) Laplacian matrix. They found that the synchronizability of scale-free network is strongly enhanced when $\beta=1$. Comparing Eq. (1) with Eq. (7), it is easily found that their optimal parameter value $\beta=1$ is equivalent to the best strategy $\alpha=-1$ in our first model. Another common character in both our works is that only the interaction of coupling oscillators or the routing regularity exists between neighbor nodes. Generally speaking, the process of synchronization can be considered as the signal transfer among the oscillators which is also correlated with traffic. Suppose that if much information of synchronized state pass through a node, it will tend to get overloaded on the node and lead to loss of information [40]. This phenomenon corresponds to the congestion in traffic systems. Due to the above common features, we can conclude that perhaps synchronizability and traffic handling capacity of the complex network are mainly determined by the same factor which is worthwhile to be explored by further efforts of physics community.

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