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Efficient routing on scale-free networks based on local information

Chuan-Yang Yin, Bing-Hong Wang, Wen-Xu Wang, Tao Zhou *, Hui-Jie Yang

Department of Modern Physics and Nonlinear Science Center, University of Science and Technology of China, Hefei 230026, PR China

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Abstract

In this Letter, we propose a new routing strategy with a single tunable parameter α only based on local information of network topology. The probability that a given node i with degree k_i receives packets from its neighbors is proportional to k_i^α. In order to maximize the packets handling capacity of underlying structure that can be measured by the critical point of continuous phase transition from free flow to congestion, the optimal value of α is sought out. Through investigating the distributions of queue length on each node in free state, we give an explanation why the delivering capacity of the network can be enhanced by choosing the optimal α. Furthermore, dynamic properties right after the critical point are also studied. Interestingly, it is found that although the system enters the congestion state, it still possesses partial delivering capability which does not depend on α. This phenomenon suggests that the capacity of the scale-free network can be enhanced by increasing the forwarding ability of small important nodes which bear severe congestion.

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Since the seminal work on the small-world phenomenon by Watts and Strogatz [1] and scale-free networks by Barabási and Albert [2], the evolution mechanism of the structure and the dynamics on the networks have recently generated a lot of interest among physics community [3,4]. One of the ultimate goals of the current studies on complex networks is to understand and explain the workings of systems built upon them [5–13], and relatively, how the dynamics affect the network topology [14–17]. We focus on the traffic dynamics upon complex networks, which can be applied everywhere, especially the vehicle flow problem on highway networks and the information flow dynamic on interconnection computer networks. Some previous works have focused on finding the optimal strategies for searching target on the scale-free networks [18] and others have investigated the dynamics of information flow with respect to the packets handling capacity of the communication networks [19–25], however, few of which incorporate these two aspects. In this Letter, we address a new routing strategy based on the local information in order to maximize the capacity of huge communication networks.

In order to obtain the shortest path between any pair of nodes, one has to know the whole network structure completely. However, the huge size of the modern communication networks and continuous growth of their structure make it usually an impossible task. Even though the networks are fixed, for the sake of routing packets along the shortest path, each node has to put all the shortest paths between any pair of nodes into its routing table, which is also impractical for huge network size because of the limitation of node storage capacity. Therefore, In contrast to previous works allowing the data packets forwarding along the shortest path, in our model, we assume each node only has the topology knowledge of it's neighbors. For simplicity, we treat all nodes as both hosts and routers for generating and delivering packets. The node capacity C, that is the number of data packets a node can forward to other nodes at each time step, is also assumed to be a constant for simplicity. In this letter, we set $C = 10$.

Recent studies indicate that many communication networks such as Internet and WWW are not homogeneous like random

* Corresponding author.
E-mail addresses: bhwang@ustc.edu.cn (B.-H. Wang), zhutou@ustc.edu (T. Zhou).

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or regular networks, but the probability that a given node has k connections to other nodes follows a power law $P(k) \sim k^{-\gamma}$ with $2 \leqslant \gamma \leqslant 3$. Barabási and Albert proposed a famous simple model (BA for short) called scale-free networks [2], of which the degree distribution is in good accordance with real observation of communication networks. We construct the network structure following the same method used in Ref. [2]: starting from m fully connected nodes, a new node with m links is added to the existing graph at each time step according to the rule of preferential attachment, i.e., the probability of being connected to an existing node is proportional to the degree of that node. Here, we choose $m = 5$ and network size $N = 1000$ fixed for simulations. Then our traffic model is described as follows: at each time step, there are R packets generated in the system, with randomly chosen sources and destinations, and all nodes can deliver at most C packets toward their destinations. To navigate packets, each node performs a local search among its neighbors. If a packet's destination is found within the searched area, it will be delivered directly to its target, otherwise, it will be forwarded to a neighbor j of node i according to the probability

$$\Pi_{i\to j} = \frac{k_j^\alpha}{\sum_l k_l^\alpha}, \tag{1}$$

where the sum runs over the neighbors (searched area) of node i and α is an adjustable parameter. Once a packet arrives at its destination, it will be removed from the system. We should also note that the queue length of each node is assumed to be unlimited and the FIFO (first in first out) discipline is applied at each queue [23]. Another important rule called path iteration avoidance (PIA) is that a link between any pair of nodes is not allowed to be visited more than twice by the same packet. Without this rule, the capacity of the network is quite low due to many times' unnecessary visiting to the same links by the same packets, which does not exist in the real traffic systems. We note that PIA does not damage the advantage of local routing strategy. If each packet records the links it has visited, this rule can be easily implemented.

One of the most interesting properties of traffic system is the packets handling and delivering capacity of the whole network. As a remark, there is difference between the capacity of network and nodes. The capacity of each node is set to be constant. While the capacity of the entire network is measured by the critical generating rate R_c at which a continuous phase transition will occur from free state to congestion. The free state refers to the balance between created packets and removed packets at the same time. When the system enters the jam state, it means packets continuously accumulate in the system and finally few packets can reach their destinations. In order to describe the critical point accurately, we use the order parameter [20]:

$$\eta(R) = \lim_{t\to\infty} \frac{C}{R} \frac{\langle \Delta N_p \rangle}{\Delta t}, \tag{2}$$

where $\Delta N_p = N(t + \Delta t) - N(t)$ with $\langle \cdots \rangle$ indicates average over time windows of width Δt and $N_p(t)$ represents the number of data packets within the networks at time t. For $R < R_c$, $\langle \Delta N_p \rangle = 0$ and $\eta(R) = 0$, indicating that the system is in the free state with no traffic congestion. Otherwise for $R > R_c$, $\eta \to r$, where r is a constant larger than zero, the system will collapse ultimately. As shown in Fig. 1, the order parameter versus generating rate R by choosing different value of parameter α is reported. It is easy to find that the capacity of the system is not the same for different α, thus, a natural question is addressed: what is the optimal value of α for maximizing the network's capacity? Simulation results demonstrate that the optimal performance of the system corresponds to $\alpha \approx -1$ (see Fig. 2). Compared to previous work by Kim et al. [18], one of the best strategies is PRF (preferential choice, which means the node with the larger degree has the higher probability to receive packets) corresponding to our strategy with $\alpha = 1$. By adopting this strategy a packet can reach its target node most rapidly without considering the capacity of the network. This result may be very useful for search engine such as google, but for traffic systems the factor of traffic jam cannot be ne-

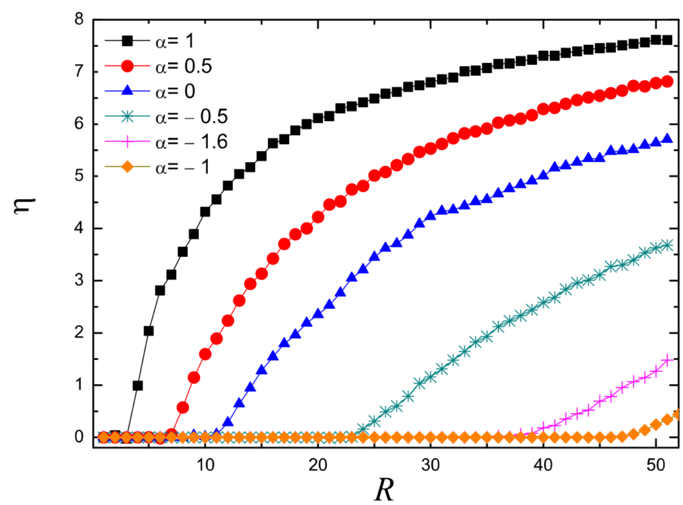

Fig. 1. The order parameter η versus R for BA network with different free parameter α. According to Eq. (2), η is calculated from 10 000 time steps. (Color online.)

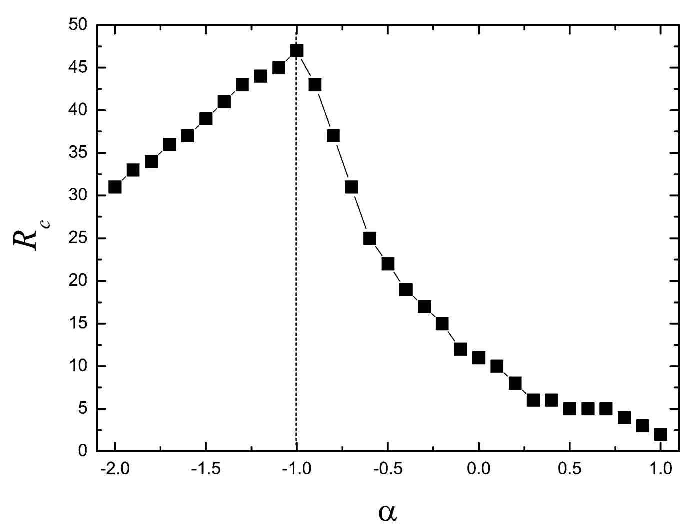

Fig. 2. The critical R_c versus α. The maximum of R_c corresponds to $\alpha = -1$ marked by dot line.

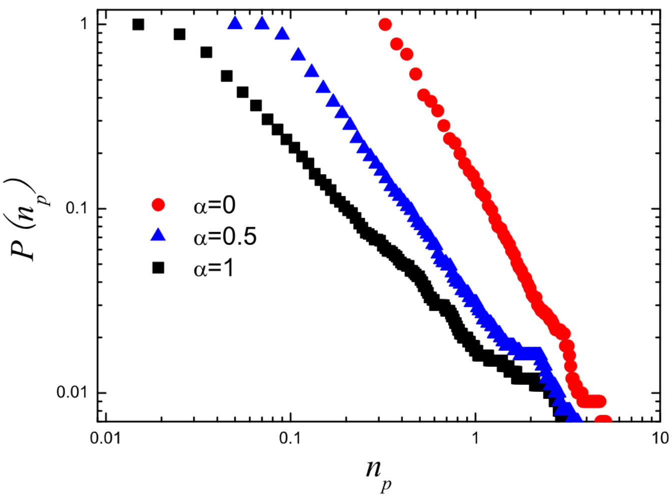

Fig. 3. The queue length cumulative distribution on each node by choosing different α more than zero. Data are consistent with power-law behavior. (Color online.)

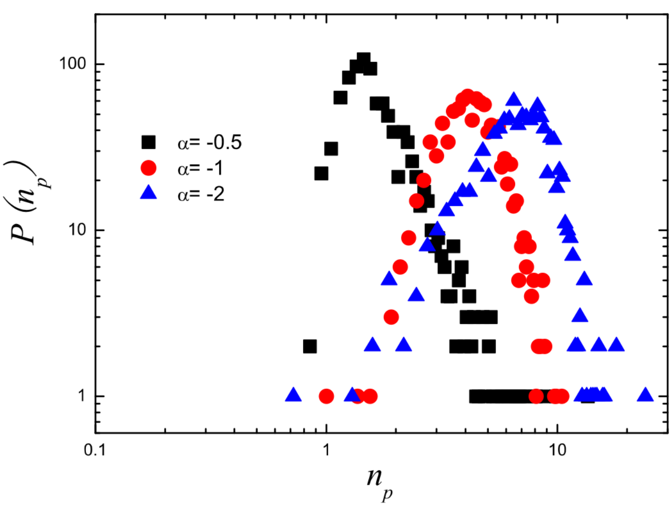

Fig. 4. The queue length cumulative distribution on each node by choosing different α less than zero. $P(n_p)$ approximately exhibits a Poisson distribution. (Color online.)

glected. Actually, average time for the packets spending on the network can be also reflected by the system capacity. It will indeed reduce the network's capacity if packets spend too much time before arriving at their destinations.

To better understand why $\alpha = -1$ is the optimal choice, we also investigate the distribution of queue length on each node with different α in the stable state. Fig. 3 shows that when $\alpha \geqslant 0$, the queue length of the network follows the power-law distribution which reflects high heterogeneous traffic on each node. Some nodes with large degree bear severe traffic congestion while other nodes hold few packets. This heterogeneous behavior is more obviously corresponding to the slope reduction with α increased from zero. But due to the same delivering capacity of all nodes, this phenomenon will undoubtedly do harm to the system capacity because of the severe overburden of small quantities of nodes. In contrast to Fig. 3, Fig. 4 shows better condition of the networks with queue length approximately displaying the Poisson distribution which represents the homo-

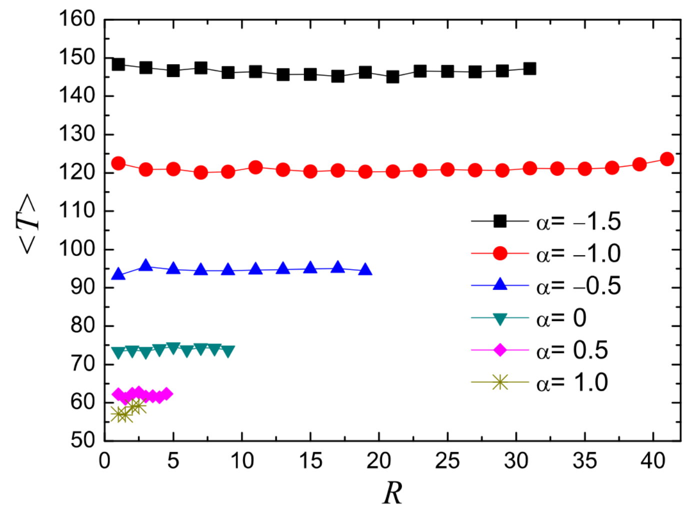

Fig. 5. Average packets travel time $\langle T \rangle$ as a function of R for different α in the case of $R < R_c$. (Color online.)

geneous of each node like the degree distribution of random graph. From this aspect, we find that the capacity of the system with $\alpha < 0$ is larger than that with $\alpha > 0$. But it's still not the whole story, in fact, the system's capacity is not only determined by the capacity of each node, but also by the actual path length of each packet from its source to destination. Supposing that if all packets bypass the nodes with large degree, it will also cause the inefficient routing for ignoring the important effect of hub nodes on scale-free networks. By the competition of these two factors, the nontrivial value $\alpha = -1$ is obtained corresponding to the maximal network's capacity. In addition, we note that according to Eq. (1), the present strategy has no effect on homogeneous networks, such as random and regular networks.

The efficiency of the system can be characterized not only by the network capacity but also by the packets transmission speed. We investigate the average packets travel time $\langle T \rangle$ as a function of R for different α in the free flow state. The packet travel time is defined as the time that a packet spends from its origin to destination. The average is taken over large quantities of packets for a long period $t = 10\,000$. As shown in Fig. 5, one can find that in the steady state, $\langle T \rangle$ is almost independent of R. This behavior can be explained by the facts that the routing strategy is only related with the topological information and no congestion occurs in the steady state. Moreover, one can find that the larger the value of α, the shorter the $\langle T \rangle$. The short $\langle T \rangle$ is ascribed to the hub effect of large degree nodes. For large α, the packets tend to be delivered to the large degree nodes, and packets in those nodes with large searched area can find their destinations with large probability. Therefore, increasing α can reduce the average packets travel time, and in the case of $R < R_c$, larger α corresponds to higher transmission speed.

The behavior in jam state is also interesting for alleviating traffic congestion. Fig. 6 displays the evolution of $N_p(t)$ (i.e., the number of packets within the network) with different R. α is fixed to be -1.5 and R_c for $\alpha = -1.5$ is 39. All curves in this figure can be approximately separated into two ranges. The starting section shows the superposition of all curves which

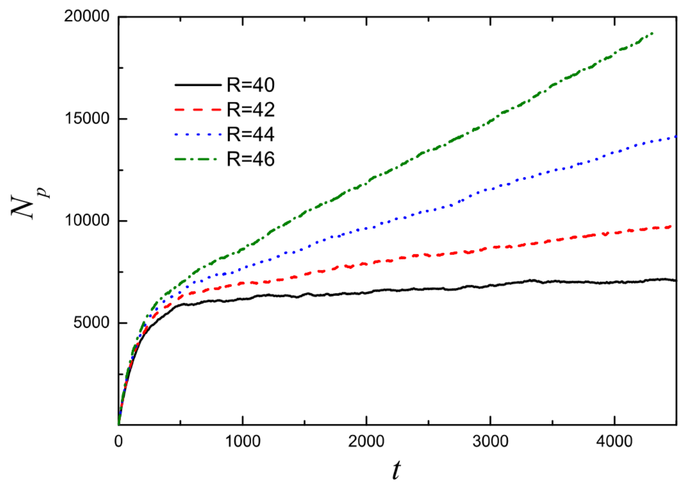

Fig. 6. The evolution of N_p for $R > R_c$. Here, α_c takes -1.5 corresponding to the critical point $R_c = 39$. (Color online.)

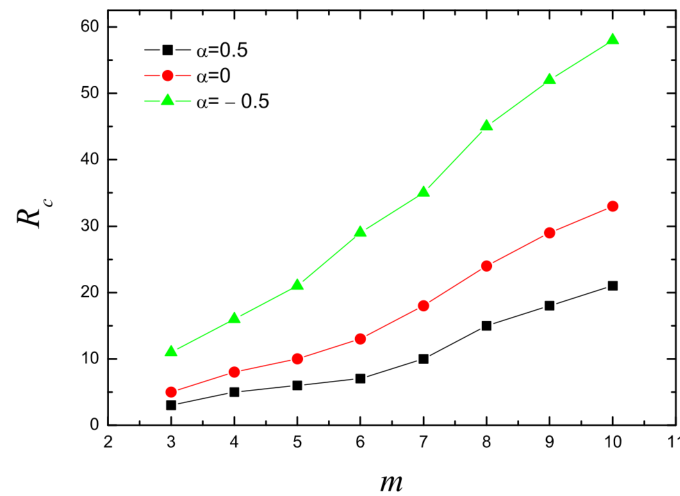

Fig. 8. The network capacity R_c as a function of m. (Color online.)

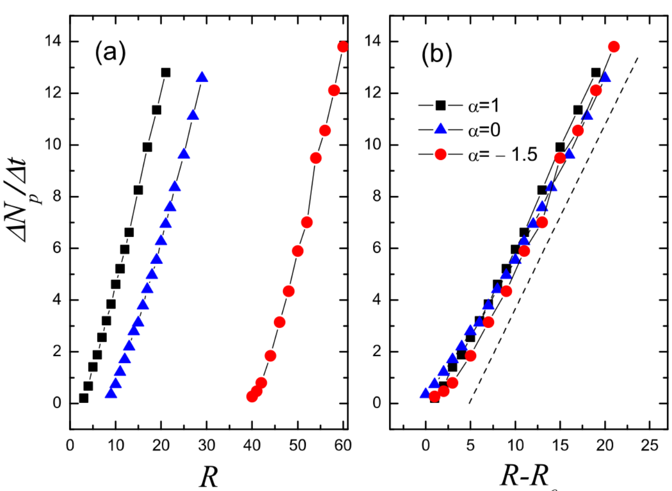

Fig. 7. The ratio between ΔN_p and time step interval Δt versus R (a) and versus $R - R_c$ the rescaling of R (b) for different α. In (b) three curves collapse to a single line with the slope ≈ 0.7 marked by a dashed line. (Color online.)

can be explained by the fact that few packets reach their destinations in a short time so that the increasing velocity of N_p is equal to R. Then after transient time, N_p turns to be a linear function of t. Contrary to one's intuition, the slope of each line is not $R - R_c$. We investigate the increasing speed of N_p depending on R by choosing different parameter α. In Fig. 7(a), in the congestion state N_p increases linearly with the increment of R. Surprisingly, after x axis is rescaled to be $R - R_c$, three curves approximately collapse to a single line with the slope ≈ 0.7 in Fig. 7(b). On one hand, this result indicates that in the jam state when R is not too large, the dynamics of the system do not depend on α. On the other hand, the slope less than 1 reveals that not all the $R - R_c$ packets are accumulated per step in the network, but about 30 percent packets do not pass through any congested nodes, thus they can reach their destinations without contribution to the network congestion. This point also shows that when R is not too large in the congestion state, the congested nodes in the network only take the minority, while most other nodes can still work. Therefore, the congestion of the system can be alleviated just by enhancing the processing capacity of a small number of heavily congested nodes. Furthermore, we study the critical point R_c affected by the link density of BA network. As shown in Fig. 8, increment of m considerably enhances the capacity of BA network measured by R_c due to the fact that with high link density, packets can more easily find their target nodes.

Motivated by the problem of traffic congestion in large communication networks, we have introduced a new routing strategy only based on local information. Influenced by two factors of each node's capacity and navigation efficiency of packets, the optimal parameter $\alpha = -1$ is obtained for maximizing the whole system's capacity. Dynamic behavior such as increase velocity of N_p in the jam state shows the universal properties which do not depend on α. In addition, the property that scale-free network with occurrence of congestion still possesses partial delivering ability suggests that only improving processing ability of the minority of heavily congested nodes can considerably enhance the capacity of the system. The critical value R_c depending on m is also investigated. Our study may be useful for designing communication protocols for large scale-free communication networks due to the local information the strategy only based on and the simplicity for application. The results of current work also shed some light on alleviating the congestion of modern technological networks.

Finally, it is worthwhile to emphasize that this PIA routing algorithm does not damage the advantage of local routing strategy. If each packet records the links it has visited, the PIA can be easily performed. One can find that this rule does not need the global topological information. Therefore, we think this rule is reasonable and can considerably enhance the network capacity. By the way, because of the homogeneity of the regular and random networks, our routing strategy has no effect on these two kinds of networks and the packets moves like a random walk. The process of random walk has been analyzed theoretically, wherein the probability of a packets found in a given node is proportional to the degree of that node, and this result is independent of the networks structure [26].

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