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# TRAFFIC DRIVEN MODEL FOR WEIGHTED NETWORKS

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**Abstract.** We propose an evolutionary weighted model for technological networks based on the interplay of traffic and infrastructure of the network by introducing a strength coupling mechanism. The model gives many statical properties such as evolution of strength, distribution of strength, degree and weight supported by empirical evidence. In particular, the model generates nontrivial degree-strength correlation observed in real-world networks. Interestingly, all properties are adjusted depending on a single parameter that represents the speed of total weight growth of the network.

**Keywords.** complex network, weighted network, traffic driven, evolution dynamics, strength-degree correlation, power-law distribution

**AMS (MOS) subject classification: 05C90, 94C15.**

# 1 Introduction

In the past few years, the studies of evolutionary dynamics in complex networks have triggered great interest from the community of physicists. Prototypical examples of real systems cover as diverse as the Internet [1], World-Wide Web [2], airport networks (AN) [3,4], protein interacting networks [5], and so on. The infrastructure of these networks exhibits many general and nontrivial properties such as small-world [6] and scale-free phenomena [7]. These interesting phenomena demonstrate that there perhaps exist some intrinsic and common mechanisms that lead to the evolution of such complex networks. In this perspective, lots of models have been proposed to explain the evolutionary dynamics of networks. Among them, two most famous models are introduced by Barabási, Albert (BA) and Watts, Strogatz (WS) characterizing the scale-free and small-world features, respectively. However, networks are far from the boolean structure, thus purely topological characterization will miss the important attributes often encountered in real-world systems. Most recently, the availability of completely weighted empirical

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evidence makes the modelling of weighted networks possible. Barrat et al. (BBV for short) presented a weighted driven model, which generates scale-free distribution of strength, degree and weight, and the exponent of the distribution is adjusted by an exclusive parameter [8]. However, the feature of strength-degree correlation obtained from this model is not in consistent with the empirical data. Therefore, there perhaps are some other mechanisms not reflected by the BBV model, which should be addressed to mimic the real-world networks' evolution.

The network evolution and the traffic on the underlying structure are usually considered to be uncorrelated. Almost all previous works did not characterize the effect of traffic increase on the evolution of the structure, just considering traffic as an appendix of the structure and study the feature of traffic on a fixed graph [9-11]. Actually, the interplay of traffic and topology is crucial to the growth of the network. Take the airline network as an example. The potential traffic demand is the key factor that triggers the building of a new line between two airports during the growth of the network. Airline is built more easily between two metropolis due to the fact of popularity and economy that can reflect the traffic demand directly. But it's not the whole story. The weights along the links will also be reinforced after the connection is established based on the development of cities. Thus, a weighted network model can describe the real world more precisely than an unweighted one.

A weighted network is often described by an weighted adjacent matrix $w_{ij}$, which represents the weight on the link between node $i$ and $j$, with $i, j = 1, \dots, N$, where $N$ is the size of the network. Here, only the undirected network with symmetric weights $w_{ij} = w_{ji}$ is considered. The definition of degree can be extended to strength naturally, as $s_i = \sum_{j \in \Gamma(i)} w_{ij}$, where the sum runs over the set $\Gamma(i)$ of neighbors of node $i$. The strength of a node integrates the information about its degree and the weights along its links, thus can characterize the importance of the node more accurately than its degree. For instance, strength in the Internet describes the actual information traffic going through a router and reflects the status of a given router. As observed in many real networks, complex networks often display a power-law degree distribution $P(k) \sim k^{-\gamma}$ with $2 \leq \gamma \leq 3$ [7]. The weight distribution $P(w)$ that denotes the proportion of the links with a given weight $w$ is another significant measure and shows a heavy tail from empirical data. Real evidence such as airline networks also indicates that strength distribution has a scale-free property, which implies a phenomenon of "reach gets reacher" [3,4]. Most resent real observations exhibit a nonlinear correlation between strength and degree of a given node $s \sim k^{\beta}$ but very few weighted models can generate this feature naturally except [4].

The preferential attachment mechanism introduced by Barabási and Albert is considered to be necessary for generating the scale-free property. Essentially, this mechanism just describes the interactions between new nodes and old nodes. In fact such interactions also exist between old nodes during the growth of the network. From this viewpoint, we express such interactions

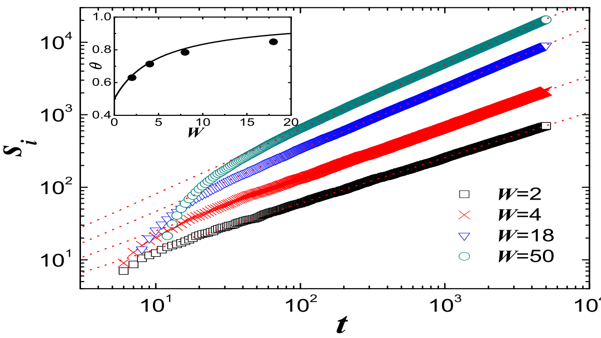


Figure 1: Evolution of strengths of nodes during the growth of networks for various parameters $W$. In the inset, we give the value of $\theta$ obtained by data fitting (filled circles), together with its analytical expression $\theta = \frac{2W+3}{2W+6}$.

among all nodes by presenting a simple and natural form $s_i s_j$, which means that the interactions between node $i$ and node $j$ is proportion to the product of their strengths. In order to unify the preferential attachment of a newly added node to the product form, we can rewrite the preferential probability as

$$\Pi^{BA}_{n \to i} = \frac{k_i}{\sum_j k_j} = \frac{k_n k_i}{\sum_j k_n k_j}, \qquad (1)$$

where $k_n$ is the degree of the newly added node. Analogously, this form can be easily extended to weighted networks, as

$$\Pi_{n \to i} = \frac{s_i}{\sum_j s_j} = \frac{s_n s_i}{\sum_j s_n s_j}. \qquad (2)$$

Thus, the interactions of the product form are unified to be a universal mechanism which exists among all nodes. This mechanism not only controls the creation of new links between nodes but also affect the increase of traffic flow along the links with the growth of the network. Our perspectives have been partly inspired by the model of Dorogovtsev and Mendes [12], who presented a class of undirected and unweighted models where the links between existing nodes can be created and removed.

## 2 The model and analytic results

The model starts with $N_0$ nodes fully connected by links with the same assigned weight $w_0$. The model is defined on two coupled mechanisms: the topological growth and the strength update rules.

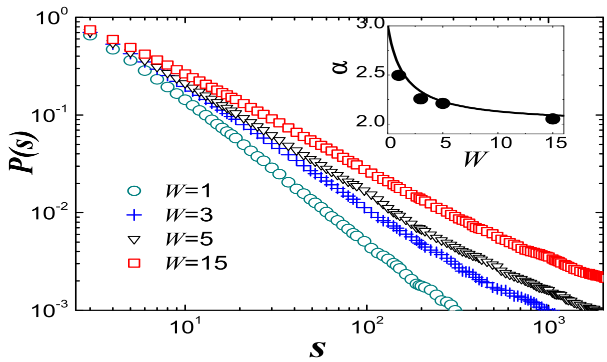


Figure 2: Probability distribution $P(s)$. Data are consistent with a power-law behavior $s^{-\alpha}$. In the inset, we give the value of $\alpha$ obtained by data fitting (filled circles), together with the analytical expression $\alpha = 2+m/(m+2W)$(line). The data are averaged over 20 networks of size N=5000.

(i) *Topological Growth.* At each time step, a new node is added with $m$ links, which is randomly attached to an existing node $i$ according to the strength preferential probability $\Pi_{n\to i}$ given in Eq.(2). The weights of all new links are set to be $w_0$.

(ii) *Strength Update Rules.* From the beginning of the evolution, any pair of nodes with or without links at each time step are allowed to create new links (if without links) or update the weights of the existing links according to the following strength-coupling mechanism:

$$w_{ij} \to \begin{cases} w_{ij}+1, & \text{with probability } Wp_{ij} \\ w_{ij}, & \text{with probability } 1-Wp_{ij}, \end{cases} \tag{3}$$

where

$$p_{ij} = \frac{s_i s_j}{\sum_{a<b} s_a s_b}. \tag{4}$$

Integrates the coupling strength of nodes $i$ and $j$, and control the evolution of weights $w_{ij}$ (if $i$ and $j$ are unconnected, $w_{ij} = 0$). The growth of total weights of links is statistically determined by the summation $< \sum_{i<j} \Delta w_{ij} >= W$, which is assumed constant for simplicity, where $W$ reflects the growing speed of the existing network's traffic flow along all the links. The potential traffic demand reflected by parameter $W$ plays a crucial role in the evolution of the network. One may notice that $Wp_{ij}$ is very likely to exceed one if the network size is not so large. When $Wp_{ij}$ exceeds one, it is automatically assumed to be one. This performance indeed will affect the initial evolution of network structure, but we have checked that with the growing of the graph size the static properties are independent of initial states.

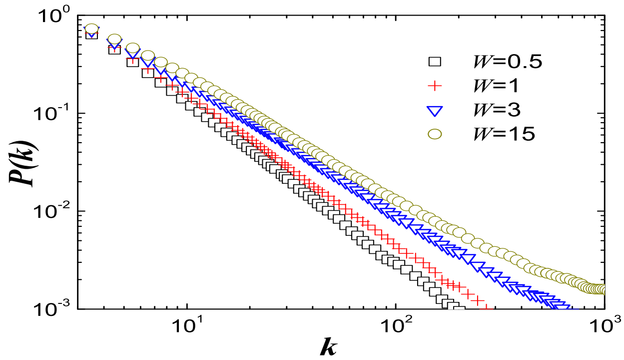


Figure 3: Probability distribution of the degrees $P(k) \sim k^{-\gamma}$. The data are averaged over 20 networks of size N=5000.

The model time is measured with respect to the number of nodes being added to the graph, i.e. $t = N - N_0$, and the natural time scale of model dynamics is the network size $N$. Using a continuous approximation, we can treat $k$, $w$, $s$ and the time $t$ as continuous variables. Therefore, the link weight $w_{ij}$ is updated as the following evolution equation:

$$\frac{dw_{ij}}{dt} = \frac{2W s_i s_j}{\sum_{a,b(a \neq b)} s_a s_b} = \frac{2W s_i s_j}{\sum_a s_a \sum_{b(\neq a)} s_b}. \quad (5)$$

The strength $s_i$ of node $i$ can increase due to the newly added node or the weight update of any possible connections to $i$ of the existing network, thus the rate equation of strength $s_i$ can be written as

$$\frac{ds_i}{dt} = \sum_j \frac{dw_{ij}}{dt} + \frac{ms_i}{\sum_l s_l} = \frac{\sum_{j(\neq i)} 2W s_i s_j}{\sum_a s_a \sum_{b(\neq a)} s_b} + \frac{ms_i}{\sum_l s_l} = \frac{2W + m}{2W + 2m} \frac{s_i}{t}, \quad (6)$$

where the last expressions are obtained by $\sum_i s_i \approx 2(m + W)t$. Eq. (6) can be readily integrated with initial condition $s_i(t = i) = m$, yielding

$$s_i(t) = m \left(\frac{t}{i}\right)^{\frac{2W+m}{2W+2m}}. \quad (7)$$

The knowledge of the time evolution of various quantities allows us to calculate their statistical features. In fact, the time $t_i = t$ at which node $i$ enters the network is uniformly distributed in [0,t] and the strength probability distribution can be written as

$$P(s,t) = \frac{1}{t + N_0} \int_0^t \delta(s - s_i(t)) dt_i, \quad (8)$$

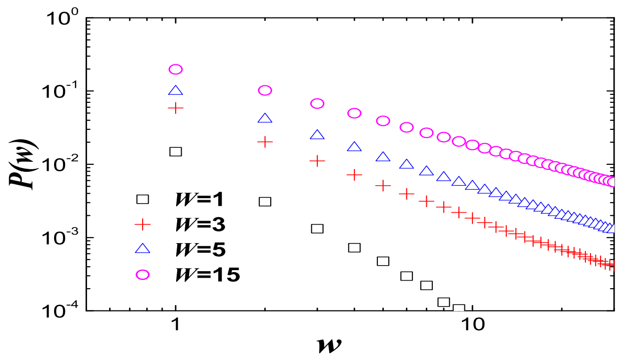


Figure 4: Probability distribution of the weights $P(w) \sim w^{-\eta}$. The data are averaged over 20 networks of size N=5000.

where $\delta(s)$ is the Dirac delta function. Using equation $s_i(t) \backsim (\frac{t}{i})^{\theta}$ obtained from Eq. (7), one obtains in the infinite size limit $t \to \infty$ the distribution $P(s) \backsim s^{-\alpha}$ with $\alpha = 1 + \frac{1}{\theta} = 2 + \frac{m}{m+2W}$. Obviously, when $W = 0$, the Barabási-Albert model is recovered with the value $\alpha = 3$. For large values of $W$, the distribution gradually becomes broader with $\alpha \to 2$ when $W \to \infty$. Taking into account the relation of strength and weight from Eq. (5), it is possible to acquire theoretical predictions for the evolution of weights and the relative statistical distribution. Combining $s_i(t) = m(\frac{t}{i})^{\frac{2W+m}{2W+2m}}$ with Eq. (5), one can obtain the rate equation of weights as follows:

$$\frac{dw_{ij}}{dt} \backsim \frac{2Wt^{2\theta}}{4(W+m)^2t^2} = \frac{W}{2(W+m)^2}t^{2(\theta-1)}. \tag{9}$$

Then, one can get the evolution of weight $w_{ij}$ by integrating Eq. (10). Similar to the previous calculation of strength distribution, weight distribution also displays a power law, $P(w) \backsim w^{-\eta}$, where $\eta = 2 + \frac{m}{W}$.

## 3 Simulation results

We report numerical simulations of networks generated by choosing different values of parameter $W$, while $N_0 = 3$, $m = 3$, and $W_0 = 1$ are fixed. We have checked that the scale-free properties of our model are independent of initial conditions. Simulation results are in good agreement with theoretical predictions. In Fig. 1, we report the behavior of the nodes' strengths versus time steps for different parameter $W$, recovering the performance predicted by analytical results. Fig. 2 shows the probability distribution $P(s) \backsim s^{\alpha}$, which is in excellent agreement with theoretical predictions. The scale-free

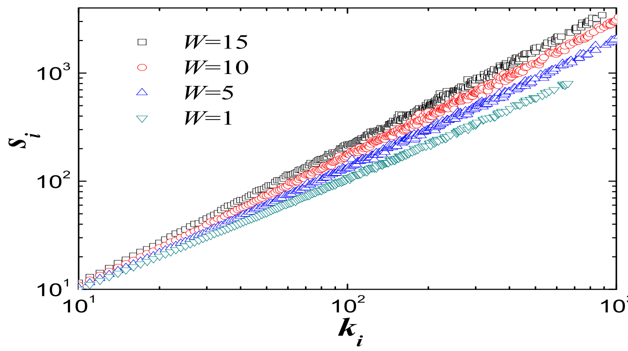


Figure 5: Strength $s_i$ versus $k_i$ for different $W$ (log-log scale). Linear data fitting gives slope 1.04, 1.17, 1.25 and 1.30 (from bottom to top), demonstrating the correlation of $s \sim k^{\beta}$.

properties of degree and weight distributions obtained from simulation results are displayed in Fig. 3 and Fig. 4, respectively. We also show the average strength $s_i$ of node $i$ with its degree $k_i$, which exhibits a nontrivial correlation $s \backsim k^{\beta}$ as confirmed by empirical observations. Unlike the BBV model (where $\beta = 1$), the exponent $\beta$ here varies with parameter $W$ in a nontrivial way as shown in Fig. 5. The correlation of $s \backsim k^{\beta}$ demonstrates a significant part of weight increment along links. More importantly, one could check the power law feature of the degree distribution $P(k) \backsim k^{-\gamma}$ by combining $s \backsim k^{\beta}$ with $P(s) \backsim s^{-\alpha}$. Considering the conservation of probability, i.e.,

$$\int_0^{\infty} P(k)dk = \int_0^{\infty} P(s)ds, \tag{10}$$

we can easily calculate the exponent $\gamma$ as follows:

$$P(k) = P(s)\frac{ds}{dk} = s_{-\alpha}\beta k_{\beta-1} = \beta k_{-\beta(\alpha-1)+1}, \tag{11}$$

obtaining $\gamma = \beta(\alpha - 1) + 1$. The simulation consistence of scale-free properties indicates that our model can indeed produce power law behaviors of degree, weight and strength, as observed in real world examples.

# 4 conclusion

We have presented a new weighted model for technological networks based on the interplay of traffic flow and network topology. The fundamental mechanism of the model can be explained from the aspect of self-organization and by realizing the function of networks. The key factor that determines the growth of the networks is the potential traffic demand. In order to satisfy the traffic demand increment, the networks' structure has to be extended. Due to the facts mentioned above, depending on a single parameter $W$, our model provides a wide variety of scale-free behaviors supported by empirical evidence. Our model may shed some new lights allowing more specific mechanisms to be integrated into better models.

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