Modeling the coevolution of topology and traffic on weighted technological networks

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For many technological networks, the network structures and the traffic taking place on them mutually interact. The demands of traffic increment spur the evolution and growth of the networks to maintain their normal and efficient functioning. In parallel, a change of the network structure leads to redistribution of the traffic. In this paper, we perform an extensive numerical and analytical study, extending results of Wang *et al.* [Phys. Rev. Lett. **94**, 188702 (2005)]. By introducing a general strength-coupling interaction driven by the traffic increment between any pair of vertices, our model generates networks of scale-free distributions of strength, weight, and degree. In particular, the obtained nonlinear correlation between vertex strength and degree, and the disassortative property demonstrate that the model is capable of characterizing weighted technological networks. Moreover, the generated graphs possess both dense clustering structures and an anti-correlation between vertex clustering and degree, which are widely observed in real-world networks. The corresponding theoretical predictions are well consistent with simulation results.

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I. INTRODUCTION

Many real-world systems existing in both nature and society can be described by complex networks. Prototypical examples cover systems as diverse as the Internet [1,2], the World Wide Web [3,4], scientific collaboration networks [5,6], protein-protein interaction networks, and worldwide airport networks [7–10]. In the past few years, much empirical evidence has revealed that small-world [11] and scale-free [12] topological properties are shared by many real networks, which triggered continuous interest from the scientific community [13,14]. Understanding and characterizing the evolutionary dynamics inducing the universal structural features in complex networks become a central issue. Many evolutionary models aiming at reproducing those common topological properties have been proposed. In addition, much effort has been devoted to studying how the dynamical processes taking place on complex networks are influenced by their structures [15–17], which has been considered as the ultimate goal of studying complex networks.

Since the first scale-free network model presented by Barabási and Albert (BA) [12], much attention has been given to modeling to better mimic the evolutionary processes of real-world networks [13]. So far, the research on networks has been primarily focused on unweighted networks [13,14], i.e., edges among vertices are either present or absent, represented as binary states. Recently, more and more real observations have indicated that the connections in many real networks are far beyond Boolean representations which would miss some important physical characters on edges. Take the Internet for example [1,2]. The load of information traffic along edges or through routers can reflect the importance of edges or routers in traffic transportation. Similarly, the number of passengers in airport networks can directly image the status of airlines [8,9]. In scientific collaboration networks, the number of papers coauthored by two scientists is a reflection of their research relationship [5,6]. The interaction strength in predator-prey networks is crucial for the stability of the ecosystem [18,19]. Therefore, there is an obvious need for a model approach that can capture the physical characteristics missing in pure network structures. Fortunately, weighted network representations provide a proper access to investigate the evolution of network structures and the weight of interactions among vertices. Moreover, recent, more complete empirical data support the study of the evolution of weighted networks. Many interesting phenomena have been observed in the analysis of real data, including a large heterogeneity in the capacity and intensity of connections, and, in particular, nonlinear correlations between weight and topology in a variety of real-world networks [7–9,20].

Very recently, Barrat, Barthélemy, and Vespignani (BBV) presented an evolutionary model to investigate weighted networks [21]. It has been deemed to be the first weighted network model that can reproduce the heterogeneity in the intensity of connections and vertices, and in the topology as well, by coupling the dynamical evolution of topology and weights [21]. Enlightened by BBV's remarkable work, many models have been presented for better mimicking real weighted networks [22–30]. We propose a traffic-driven model to investigate the coevolution of traffic and topology on weighted technological networks. Our proposal is partially inspired by noting that the BBV model takes into account only the evolution of weight and topology between newly added vertices and old ones. We argue that such interaction also exists among the old nodes. Moreover, due to the absolute difference between social and technological networks in the behavior of assortative mixing [31–33], these two types of network should be investigated individually. What underlying mechanism results in such essential difference still remains unclear [34], although some previous models did give some possible explanations [35]. We focus on the evolutionary dynamics of technological networks in the present work. We deem that traffic is ubiquitous in technological networks, and the networks self-organize and grow to maintain their functioning in routing traffic. The evolution driven by traffic occurs both between newly added vertices and existing ones and between existing vertices themselves. This is partially motivated by the work of Dorogovtsev and Mendes [36,37], who presented a class of undirected and unweighted models in which edges can be either created or removed among old vertices. Our model is fully studied based on both simulation and theoretical predictions (a short report of the model appeared in Ref. [38]). The obtained diversity of scale-free characteristics, the nontrivial clustering coefficient, the assortative behavior of strength correlation, the nonlinear strength-degree correlation, and the correlation between vertex clustering and degree have all been empirically observed, demonstrating the validity of our microscopic mechanisms.

The paper is organized as follows. In Sec. II, we review definitions of tools for investigating weighted networks. The evolutionary rules of the model are described in Sec. III. Section IV provides the theoretical predictions of the distributions of strength, degree, and weight, as well as the strength-degree correlation. In Sec. V, simulation results of evolutionary properties and distributions are reported. In Sec. VI, we discuss the strength correlation and hierarchical backbone structures. In Sec. VII, the work is summarized.

II. DEFINITIONS

The topological as well as weighted properties can be completely described by a weighted adjacency matrix W, whose elements w_{ij} denote the weight on the edge between vertex i and j. $w_{ij}=0$ represents that vertices i and j are disconnected. Here, we focus on the cases of undirected graphs, where the weights are symmetric ($w_{ij}=w_{ji}$).

In weighted networks, a natural generalization of degree is the vertex strength described as $s_i=\sum_{j\in\Gamma(i)}w_{ij}$, where the sum runs over the set $\Gamma(i)$ of neighbors of the vertex i. The physical content of strength can be easily explained. For instance, in the Internet and the worldwide airport network, s_i denotes the total traffic load passing through a router [1,2,32] or an airport at one unit time [8,9]. For scientific collaboration networks, s_i denotes the number of papers of a scientist coauthored with others [5].

Statistical properties of weighted networks can be characterized by the distributions of strength $P(s)$ and weight $P(w)$, which denote the probability of a vertex to have strength s and of an edge to have weight w. Much empirical evidence has demonstrated that the distributions of strength and weight also follow heavy-tailed distributions [8,9,17], which indicates that the heterogeneous behavior is a universal feature of weighted networks, and $\langle s\rangle$ and $\langle w\rangle$ are not typical in weighted graphs. Of particular interest is the correlation between the vertex strength and the degree, which is encoded in the statistical properties of these distributions. Previously reported results have displayed that there exists a nonlinear relationship between strengths and degrees $s\sim k^{\alpha}$ with $\alpha>1$ [8,9,17], implying a "rich get richer" effect, i.e., large-degree vertices usually afford higher strength.

A high clustering coefficient is a common property of real-world networks, ranging from society to nature [11,13,15]. The clustering coefficient is used to quantify the emergence of cliques, representing circles of friends or acquaintances in which every member knows every other member. In addition, a power correlation has been widely observed between vertex clustering and degree with exponent -1, i.e., $C(k)\sim k^{-1}$ [39].

The assortative mixing property has gained much attention in complex networks [31–33]. In social networks, connections between people are usually assortative, while for other types of networks, like technological and biological networks, vertices in the network tend to form connections to others unlike them, i.e., disassortative behavior. The simplest and most intuitive method to measure the correlation of vertex degrees is the average nearest-neighbor degree [32,40]. Analogously, the average nearest-neighbor strength of vertex i can be defined as

$$s_{NN,i}=\frac{1}{k_i}\sum_j a_{ij}s_j. \tag{1}$$

Once averaged over classes of vertices with strength s, the average nearest-neighbor strength can be expressed as

$$s_{NN}(s)=\sum_{s'} s'P(s'|s), \tag{2}$$

providing a probe of the strength correlation function. Here, $P(s'|s)$ denotes the conditional probability that an s-strength vertex connects to an s'-strength neighboring vertex.

III. THE MODEL

Previous models for weighted networks gave us the understanding that the network growth is driven by topological features with weights only statically assigned to the edges, or coupled the evolution of weights and topologies spurred only by newly added vertices [21,41,42]. We argue that for technological networks traffic (denoted by weights) is an intrinsic character and traffic-driven interactions should exist not only between newly added vertices and old ones, but also in the whole network, i.e., among the existing vertices. Perhaps the most reasonable and the simplest way to express such reinforcement of interaction among edges is the product form of related vertex strengths, i.e., the pairwise interaction between vertices i and j is proportional to s_is_j (strength-coupling form). Let us review the BA model [12]: a new vertex n with m edges is attached at each time step in such a way that the probability Π_i of being connected to the existing node i is proportional to the degree k_i of node i, which can be written in the product form of degrees:

$$\Pi^{BA}_{n\to i}=\frac{k_i}{\sum_j k_j}=\frac{k_nk_i}{\sum_j k_nk_j}. \tag{3}$$

Similarly, in the BBV model, one can rewrite the strength preferential probability as

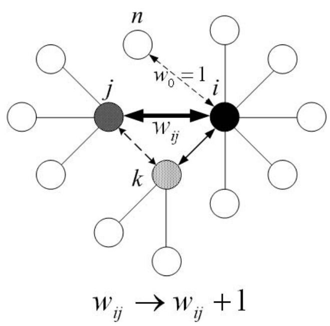

FIG. 1. Illustration of the evolution dynamics. A new node n connects to a node i with probability proportional to $s_i=\Sigma_j s_j$. The thicknesses of nodes and links, respectively, represent the magnitudes of the strength and weight. New connections (dashed lines) can be built between preexisting nodes, and the bilateral links represent the traffic growing process along links under the general mechanism of strength couplings.

$$\Pi_{n\to i} = \frac{s_i}{\sum_j s_j} = \frac{s_n s_i}{\sum_j s_n s_j}. \tag{4}$$

Actually, such interaction should exist among old vertices in the same way, including the updating of weights along edges and creating new edges between old vertices. It should be noted that edge creation can also be considered as the reinforcement of weights from zero to nonzero.

Our model consists of two evolution mechanisms: the strength dynamics and the topological growth (see Fig. 1). We start from an initial configuration of N_0 vertices connected by edges with assigned weight $w_0=1$.

(i) Strength dynamics. From the beginning of the evolution, all the possible connections at each time step are allowed to update their weights according to the strength-coupling mechanism:

$$w_{ij} \to \begin{cases} w_{ij}+1 & \text{with probability } Wp_{ij}, \\ w_{ij} & \text{with probability } 1-Wp_{ij}, \end{cases} \tag{5}$$

where

$$p_{ij} = \frac{s_i s_j}{\sum_{a<b} s_a s_b} \tag{6}$$

couples the strength of vertices i and j and governs the incremental probability of weight w_{ij} (if i and j are disconnected, $w_{ij}=0$). The total weight of the edges in a statistical sense is controlled by the amount $\langle \Sigma_{i<j}\Delta w_{ij}\rangle = W$, which is assumed to be consistent for simplicity. The exclusive parameter W reflects the growing speed of the total traffic load in the network. For the Internet and the airport network, W denotes the increasing rate of information flow and passenger flow, respectively. Continuously growing traffic plays the driving role in the network evolution. One may notice that Wp_{ij} is very likely to exceed 1 if the number of initial nodes N_0 is too small. When this happens, Wp_{ij} is assumed to be 1. This management of Wp_{ij} will have some influence on the initial evolution of the network, but it has no effect on the statistical properties, as they are nearly independent of initial state.

(ii) Topological growth. At the same time step, a new vertex n is added to the existing network with M edges that are randomly attached to an existing vertex i according to the strength preferential probability

$$\Pi_{n\to i} = \frac{s_n s_i}{\sum_j s_n s_j} = \frac{s_i}{\sum_j s_j}, \tag{7}$$

where the sum runs over all existing vertices. The weight of each new edge is set to be $w_0=1$. In fact, the strength preferential attachment is essentially the same as the mechanism of strength-coupling interactions we have discussed.

The mechanisms (i) and (ii) capture the fact that the weighted network has to be adaptive to its internal traffic increment. Take the airport network for example. Due to social economic development, there is a need to build new airports in some cities. The traffic demand from these new cities to metropolises is obviously larger than that to small cities; thus new connections will be preferentially attached to cities of higher social status and population. Perhaps the strength of an airport, representing the whole traffic load passing through it, is the most reasonable element to characterize the significance of the airport. All these phenomena can be reflected by the topology growth mechanism, which couples the evolution of the strength and the expansion of the network. In parallel, traffic flow among existing airports will keep growing, leading to the increment of weights along edges, in particular the edges between metropolises. The traffic increasing velocity of an edge is quantified by the strength-coupling form, which considers both cities' contribution to the traffic load between them. The internal traffic demand also results in the emergence of connections among old airports to alleviate traffic congestion and improve traffic efficiency. New airlines should be preferentially built between high-strength cities, because otherwise avoidable traffic jams would occur among indirect routes with connections to large cities. This internal evolution can be well captured by our strength dynamics rule which describes, by adopting the strength-coupling mechanism, the weight updating on edges as well as the preferential building of new connections among vertices, both driven by the traffic demand. Similarly, for the Internet, information flow should not be judged to be the subordinate of the network. From the viewpoint of self-organization, the evolution of the network is driven by the augmentation of information traffic in order to maintain the efficiency and normal functioning of the network.

IV. THEORETICAL PREDICTIONS OF STRENGTH, DEGREE, AND WEIGHT

In this section, we report theoretical predictions of evolutionary behavior and distributions of strength, degree, and weight.

Let $s_i(t)$ be the average weight of the ith node at time t; then s_i satisfies the following constraint:

$$\sum_j s_j = (2M+2W)t, \tag{8}$$

and the time evolution equation in the framework of the mean field approximation is

$$\frac{ds_i}{dt} = \frac{Ms_i}{\sum_j s_j} + \frac{2Ws_i\left(-s_i + \sum_j s_j\right)}{\sum_j s_j \sum_k s_k - \sum_j s_j^2} \simeq \frac{Ms_i}{\sum_j s_j} + \frac{2Ws_i \sum_j s_j}{\sum_j s_j \sum_k s_k} = \frac{(M+2W)s_i}{(2M+2W)t}. \tag{9}$$

The validity of the approximation in the above equation is proved in the Appendix. The solution of Eq. (9) can be easily obtained:

$$s_i(t) = M\left(\frac{t}{t_i}\right)^{(M+2W)/(2M+2W)}. \tag{10}$$

Thus, the strength distribution can be written as

$$P(s) = \frac{1}{t}\int_1^t \delta(s - s_i(t))dt_i, \tag{11}$$

which yields the strength distribution $P(s) \sim s^{-\beta}$, with

$$\beta = 2 + \frac{M}{M+2W}. \tag{12}$$

Let $f_{ij}(t)$ represent the average weight between the ith node and the jth node at time t; here we assume $i<j$. Then the initial condition is

$$f_{ij}(t=j-1) = 0 \tag{13}$$

and in the framework of the mean field approximation, the time evolution is described by

$$\frac{df_{ij}}{dt} = 2W\frac{s_i s_j}{\sum_k s_k \sum_l s_l}. \tag{14}$$

Substituting Eq. (10) into Eq. (14), one easily obtains that

$$f_{ij}(t) = \frac{M^2}{2M+2W}\left(\frac{t_j}{t_i}\right)^{(M+2W)/(2M+2W)}\frac{1}{t_j} + \int_{t_j}^{t} dt' \frac{M^2 2W}{(2M+2W)^2 t'^2} \times \left(\frac{1}{t_i t_j}\right)^{(M+2W)/(2M+2W)} t'^{[2(M+2W)]/(2M+2W)} = \frac{M^2}{2M+2W}\left(\frac{1}{t_i t_j}\right)^{(M+2W)/(2M+2W)} t^{2W/(2M+2W)}. \tag{15}$$

The distribution of weight can be analogously obtained by

$$P(w) = \frac{1}{t^2}\int_1^t dt_i dt_j \delta(f_{ij}(t) - w), \tag{16}$$

which yields a power-law distribution of weight $P(w) \sim w^{-\theta}$ with

$$\theta = 2 + \frac{M}{M+2W}. \tag{17}$$

When $j<i$,

$$f_{ij} = f_{ji}. \tag{18}$$

To calculate the degree of the ith node at time t, we need to make the following approximation. Since f_{ij} only represents the average weight between the ith node and the jth node, the actual weight between them should be described by a distribution function. In this paper, we shall assume that this distribution function is a Poisson distribution with the average weight given by f_{ij}. Therefore, the average connectivity between the ith node and jth node is given by

$$p_{ij}(t) = 1 - e^{-f_{ij}(t)}. \tag{19}$$

(Here, the connectivity is defined as 0 when the weight between the ith node and the jth node is 0, and 1 when the weight is larger than 1.) It may be helpful to mention that when $W=0$ our model is reduced to the BA model and Eq. (19) is no longer valid. Actually, when $W=0$, the weight distribution function between i and j can have nonzero probability only for the weight of 0 or 1 and highly deviates from the Poisson distribution. However, when $W\neq 0$, $f_{ij}(t)$ is mostly contributed by the strength-coupling mechanism when $t \gg t_i, t_j$ and thus the weight distribution function can be described by the Poisson distribution function. Consequently, the average degree of the ith node at time t is

$$k_i(t) = \sum_{j=1}^{t} (1 - e^{-f_{ij}(t)}) \tag{20}$$

in comparison with the average weight

$$s_i(t) = \sum_{j=1}^{t} f_{ij}(t). \tag{21}$$

We shall calculate the difference between s_i and k_i. Since it is difficult to evaluate the term $e^{-f_{ij}}$ analytically, we shall make the following approximation:

$$x - (1 - e^{-x}) \approx = \begin{cases} x^2/2 & \text{if } x < 1, \\ x & \text{if } x \geq 1. \end{cases} \tag{22}$$

Then the difference between s_i and k_i can be expressed as

$$s_i(t) - k_i(t) \approx \sum_{j=1}^{\tau} f_{ij}(t) + \sum_{j=\tau}^{t} \frac{1}{2} f_{ij}^2(t), \tag{23}$$

where τ is given by the condition

$$f_{i\tau}(t) = 1, \tag{24}$$

which yields

$$\tau=\left(\frac{M^2}{2M+2W}\right)^{(2M+2W)/(M+2W)}\frac{1}{t_i}t^{2W/(M+2W)}. \qquad (25)$$

When

$$t_i\geq\left(\frac{M^2}{2M+2W}\right)^{(2M+2W)/(M+2W)}t^{2W/(M+2W)}, \qquad (26)$$

$\tau<1$ and

$$s_i(t)-k_i(t)=\int_1^t dt_j\frac{1}{2}f_{ij}^2(t)$$
$$\approx\frac{1}{4}\frac{M^4}{2W(M+W)}\left(\frac{t^{2W/(M+2W)}}{t_i}\right)^{[2(M+2W)]/(2M+2W)}, \qquad (27)$$

which works correctly only for $W>0$. When

$$t_i\leq\left(\frac{M^2}{2M+2W}\right)^{(2M+2W)/(M+2W)}t^{2W/(M+2W)}, \qquad (28)$$

we have

$$s_i(t)-k_i(t)=\int_1^\tau dt_jf_{ij}(t)+\frac{1}{2}\int_\tau^t dt_jf_{ij}^2(t)$$
$$=D\frac{1}{t_i}t^{2W/(M+2W)}, \qquad (29)$$

with

$$D=\frac{M(M+4W)}{4W}\left(\frac{M^2}{2M+2W}\right)^{M/(M+2W)}, \qquad (30)$$

which works correctly only for $W>0$. From the above equations, one sees that $[s_i(t)-k_i(t)]/s_i(t)\to 0$ in the large-time limit $t\gg 1$. Thus when $t\gg 1$, we expect the linear relationship $s_i=k_i$.

V. SIMULATION RESULTS OF STRENGTH, DEGREE, AND WEIGHT

The model time is measured with respect to the number of nodes added to the graph, i.e., $t=N-N_0$, and the natural time scale of the model dynamics is the network size N. From Eqs. (12), (17), and (29), one can find the distributions of strength, degree, and weight can be tuned by both parameters M and W. Thus, in the following simulations, we fix $M=3$ and adjust the main parameter W which, as mentioned earlier denotes the internal traffic increasing velocity, to study the statistical properties of the weighted network. Correspondingly, the number of initial vertices is set to be $N_0=M=3$. The initial weight of edges w_0 is fixed to be 1 for simplicity. We have checked that the scale-free properties of the generated networks are independent of initial arrangements for large network sizes.

We first investigate the evolutionary behavior of the vertex strength and weight, the difference between the two, and the difference over vertex strength. The top panel of Fig. 2 reports the strength s_i as a function of evolutionary time t,

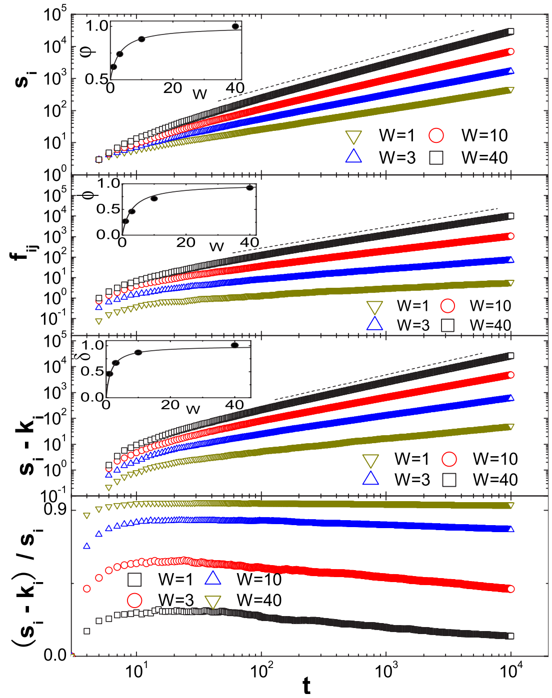

FIG. 2. (Color online) Time evolution of vertex strength s_i, weight f_{ij}, the difference of vertex strength and degree s_i-k_i, and $(s_i-k_i)/s_i$ for different values of parameter W. Here i and j are chosen as 4 and 5, respectively. Data points are obtained by averaging over 100 network realizations. The dashed lines are the fitting of the power-law behavior. The network size is 10 000. The insets are the comparisons of theoretical predictions and simulation results. The continuous curve is the analytical expression, while data points are the corresponding data fitting of the distribution.

which is the same as the network size N, since a new node is added to the network at each time step. Equation (10) has predicted that $s_i(t)$ follows a power law as time evolves, i.e., $s_i(t)\sim(t/t_i)^\varphi$ with $\varphi=(M+2W)/(2M+2W)$. Here, t_i denotes the time when vertex i enters the system. Simulations are performed by choosing $t_i=4$ for different values of parameter W with fixed $M=3$. The obtained results well display the power-law behavior for large t. In the inset of the top panel, the curve shows the theoretical predictions of φ depending on W based on the mean field theory while the data points are the correspondent numerical fittings, which match the analytical results. The second panel of Fig. 2 reports the evolution of f_{ij} by choosing $i=4,j=5$. The acquired power-law behavior well reproduces the analytical predictions from Eq. (15), i.e., $f_{ij}(t)\sim t^\phi$ with $\phi=2W/(2M+2W)$, as demonstrated in the inset. Equation (29) indicates that the difference between vertex strength and degree $s_i(t)-k_i(t)$ will obey a power law $s_i(t)-k_i(t)\sim t^\delta$ with $\delta=2W/(M+2W)$. Relevant numerical simulations are shown in the third panel of Fig. 2. The simulation results are also in good accordance with theoretical predictions as shown in the inset. The difference between vertex strength and degree divided by strength is shown in the bottom panel of Fig. 2. One can see that, for large t, $(s_i-k_i)/s_i$ is a decreasing function of t. (Moreover, according to Eqs. (10) and (29), for large W, the decreasing velocity is very slow.) Hence, when t tends to infinity, $(s_i$

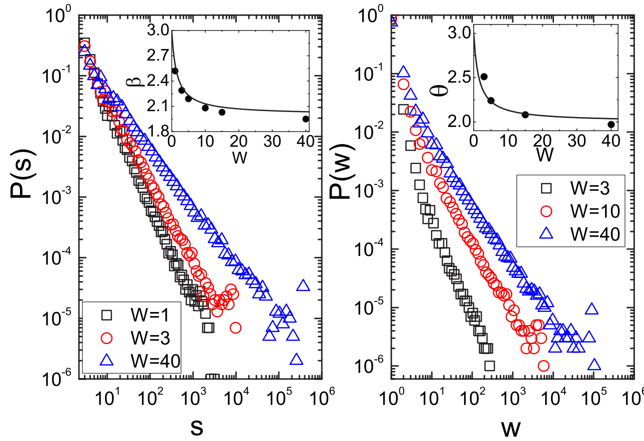

FIG. 3. (Color online) Distributions of strength (left panel) and weight (right panel) for different values of parameter W. Data points are obtained by averaging over 20 network realizations. Here we log-bin the data with bin sizes increased by multiplying with a constant factor. The network size is 40 000. The insets are the comparisons of theoretical predictions and simulation results. The continuous curve is the analytical expression; while data points are the corresponding data fitting of the distribution.

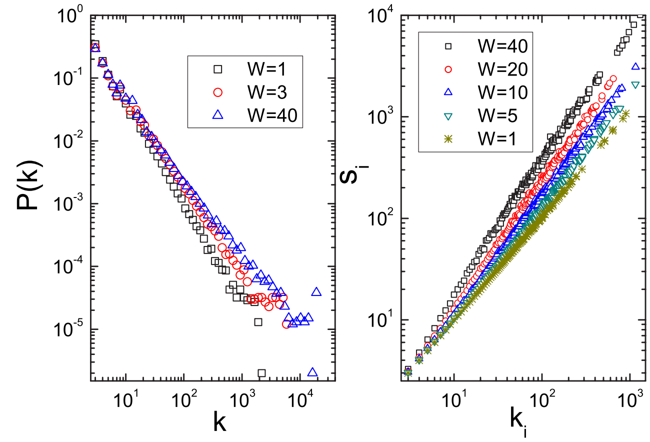

FIG. 4. (Color online) Distributions of vertex degree (left panel) and the correlation between vertex strength and degree (right panel) for different values of W. Data points are obtained by averaging over 20 network realizations. The network size is 40 000. For degree distributions, we log-bin the data with bin sizes increased by multiplying with a constant factor.

$-k_i)/s_i$ tends to zero, which is consistent with our analysis.

Below, we show numerical simulations of the statistical properties of strength, weight, and degree as well as the correlation between strength and degree, which are considered the most important statistical features of weighted networks. As shown in the left panel of Fig. 3, distributions of strength follow good power-law behavior with the exponent β depending on the value of W, which well reproduces the statistical properties of empirical data. As indicated by Eq. (12), β can be tuned in the range of 2–3, to which most real networks belong, making our model general for mimicking the evolutionary dynamics of weighted technological networks. The values of β obtained by data fitting and analytical expressions are reported in the inset of the left panel for comparison. From Eq. (12), one can find that, in the case of $W=0$, i.e., no strength dynamics, we recover the BA model with the value $\beta=3$ together with $s_i=k_i$. The weight distributions for different values of W are displayed in the right panel of Fig. 3, showing scale-free weight distributions. As predicted by Eq. (17), the tunable range of the exponent θ of the weight distributions is $2\leq\theta\leq 3$, which is the same as that of strength. The comparison between simulations and analytical predictions is reported in the inset of the right panel. Figure 3 demonstrates the validity of our theoretical predictions for the statistical distributions of strength and weight. The tunable range of the exponents of the two distributions indicates that our microscopic mechanisms can well capture the evolution of real weighted networks. (However, for a few very high-strength vertices, the distribution cannot be well predicted by mean field theory, reflected by the fluctuation in the tails.)

The simulation results for degree distributions and the correlation between vertex strength and degree are shown in Fig. 4. As shown in the right panel of Fig. 4, degree distributions follow a power law with the exponent depending on the value of the parameter W. In addition, one can find that the vertex strength s_i is a nonlinear function of corresponding degree k_i, i.e., $s_i \sim k_i^{\alpha}$ with $\alpha>1$, which is in good agreement with the real observations of the worldwide airport network. Such nonlinear correlation indicates a rich get richer phenomenon in the weighted networks, that is, a vertex of larger degree usually affords a larger traffic load in weighted technological networks.

VI. STRENGTH CORRELATION AND CLUSTERING STRUCTURE

Next, we investigate the strength correlation between a vertex and its neighbors. This quantity is a natural extension of the degree correlation. The average nearest-neighbor strength is used to measure this correlation. Relevant detailed definitions are already given in Sec. II. The strength correlation can be obtained by adopting the mean field theory

$$s_{NN}(i) = \frac{\sum_j p_{ij}(t)s_j(t)}{\sum_j p_{ij}(t)}. \tag{31}$$

Substituting Eqs. (10) and (19) into the above equation, we get

$$s_{NN}(i) = \frac{\sum_j (1-e^{-f_{ij}})M\left(\frac{t}{t_j}\right)^{(M+2W)/2(M+W)}}{\sum_j (1-e^{-f_{ij}})}, \tag{32}$$

where

$$f_{ij} = \frac{M^2}{2(M+W)}\left(\frac{1}{t_i t_j}\right)^{(M+2W)/2(M+W)} t^{2W/2(M+W)}. \tag{33}$$

Then, from Eq. (10), we can get

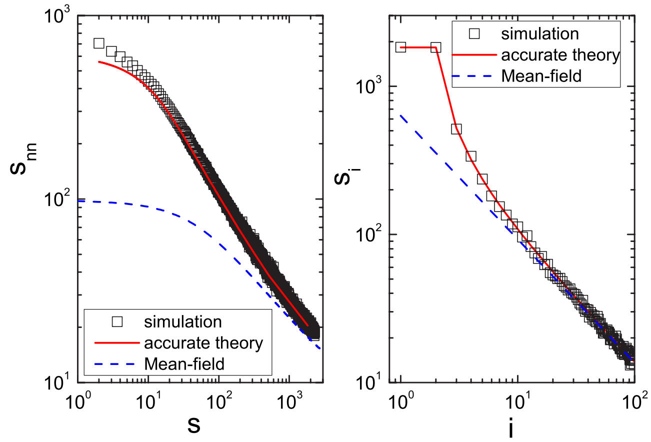

FIG. 5. (Color online) Left panel: strength correlation s_{NN} versus s. Right panel: vertex strength s_i as a function of vertex number i. The red (solid) lines are the accurate theoretical predictions and the blue (dashed) lines are the mean field theory. The parameter values are $W=4$ and $M=2$. The network size $N=1000$. Simulation results are obtained by averaging over 500 different realizations.

$$t_i = t\left(\frac{M}{s_i}\right)^{2(M+W)/(M+2W)}. \tag{34}$$

Substituting Eq. (34) into Eq. (32), we have

$$s_{NN}(s) = \frac{\sum_j (1-e^{-\mu_s})M\left(\frac{t}{t_j}\right)^{(M+2W)/2(M+W)}}{\sum_j [1-e^{-\mu_s}]}, \tag{35}$$

where

$$\mu_s = \frac{Ms}{2(M+W)}\left(\frac{1}{tt_j}\right)^{(M+2W)/2(M+W)} t^{2W/2(M+W)}. \tag{36}$$

Simulation results and corresponding analytical results of the strength correlation are shown in the left panel of Fig. 5. One can find that both the numerical and theoretical results exhibit a power-law decay in the range of large values of s, which indicates a disassortative behavior. However, the theoretical prediction differs from the simulation result despite the same qualitative trend. In order to explain this difference, we investigate the evolution of vertex strengths, i.e., $s_i(t) \sim t$. As shown in the right panel of Fig. 5, the mean field theory has predicted that $s_i(t)$ will follow

$$s_i(t) = M\left(\frac{t}{t_i}\right)^{(M+2W)/(2M+2W)} \sim i^{-(M+2W)/(2M+2W)}. \tag{37}$$

While the simulation results have shown that the strength evolution of some oldest vertices does not obey the analytical results based on the mean field theory. While, for the younger vertices ($i>100$), the simulation results are well consistent with the theoretical predictions. Since the mean field theory can predict the strength evolution of most vertices, the numerical results for distributions of strength, as well as the degree and weight, are in good agreement with the analytical results. However, for the strength correlations, those oldest vertices play a significant role and the inaccurate predictions of their strengths will have strong influence on the final results for the correlation properties, which is because the oldest vertices are usually of highest strengths and degrees, and most of their neighbors are small-degree vertices. Hence, the underestimation of the strengths of old vertices results in lower values of s_{NN} of small-strength vertices compared to the simulation results, as displayed in the left panel of Fig. 5.

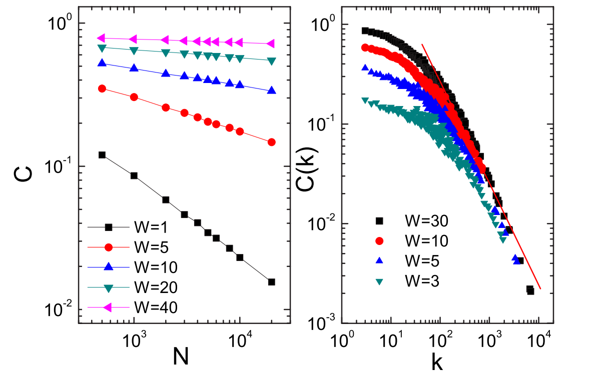

FIG. 6. (Color online) Clustering coefficient C as a function of network size N (left panel) and the dependence of vertex clustering $C(k)$ on degree k (right panel) for different values of parameter W. The line is of slope -1. Data points are obtained by averaging over 100 different realizations.

In order to well predict the correlation property, we give a more accurate theoretical method for the evolution of vertex strength. In the case of $1\leq i\leq M$, we have

$$s_i(t=1) = M, \tag{38}$$

while for $t\geq 2$,

$$s_i(t) = M\prod_{j=0}^{t-2}\left(1+\frac{M+2W}{M^2+2(M+W)j}\right). \tag{39}$$

In the case of $i>M$, we have

$$s_i(t=i-M+1) = M, \tag{40}$$

while for $t\geq i-M+2$,

$$s_i(t) = M\prod_{j=i-M}^{t-2}\left(1+\frac{M+2W}{M^2+2(M+W)j}\right). \tag{41}$$

The comparison of the accurate prediction and the simulation result is shown in the left panel of Fig. 5. Accordingly, the strength correlation can be obtained by substituting $s_i(t)$ into Eq. (31). In the left panel of Fig. 5, one can find good agreement between the theoretical prediction and simulation results.

Now we study the clustering coefficient of the network for different values of W and various network sizes. As shown in the left panel of Fig. 6, the clustering coefficient C for each value of W follows a power-law function of network sizes and higher values of W correspond to higher values of C. Moreover, the higher is the value of W, the slower the decay velocity of C. In the case of very low values of W, our model recovers the BA model, so that the generated network possesses few clustering structures. With increment of the network size, the clustering coefficient decreases to zero quickly. In contrast, for high values of W, the clustering coefficient is close to 1 and nearly independent of the network size. This indicates that our traffic-driven mechanism can effectively promote the construction of clustering structures, which are widely observed in real networks.

The correlation between the clustering and the degree of vertices is shown in the right panel of Fig. 6. One can see that, in the large-degree range, power-law behavior emerges with slope close to −1. More interestingly, our model can well mimic the flat behavior in the small-degree range observed in many real networks, such as actor networks, the semantic web, and the Internet [39]. As far as we know, few previous models can generate the power-law clustering-degree correlation together with flat head behavior. This is one of the most important properties successfully generated by our model.

VII. CONCLUSION

We have proposed a simple model by introducing universal interactions among vertices spurred by the increment of traffic demands in weighted technological networks. The model can generate a wide variety of scale-free properties of distributions of strength, degree, and weight, as well as the correlation between vertex strength and degree, which are all supported by empirical data. Theoretical predictions of these properties are also provided, which are in good accordance with the simulation results. Furthermore, we have studied the strength correlation between a vertex and its neighbors and found disassortative behavior. Corresponding analytical results are given for comparison. In addition, we have investigated the clustering coefficient and the correlation of clustering and degree of vertices. Power-law scaling and the anticorrelation between vertex clustering and degree properties are observed, which demonstrate the validity of our microscopic mechanism for modeling weighted technological networks. Due to the reproduction of all kinds of weighted and topological features of real-world weighted networks, our model may be very beneficial for future understanding or characterization of real networks.

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APPENDIX

In this appendix, we prove the validity of dropping the terms in Eq. (9). We define

$$Q(t) = \sum_{k=1}^{t} s_k^2(t), \tag{A1}$$

$$s(t) = \sum_{k=1}^{t} s_k(t) = 2(M + W)t. \tag{A2}$$

Hence, we have

$$\frac{ds_i}{dt} = \frac{(M+2W)s_i}{(2M+2W)t} + F(t)s_iQ_i(t), \tag{A3}$$

where $s_i(t=i)=M$ and

$$F(t) = \frac{2W}{s(t)[s^2(t) - Q(t)]} > 0, \tag{A4}$$

which can be obtained from the first line of Eq. (9). Define

$$Q_i(t) = Q(t) - s_i s(t). \tag{A5}$$

We can assume

$$s_1 > s_2 > \cdots > s_N, \tag{A6}$$

which yields

$$Q_1 < Q_2 < \cdots < Q_N, \tag{A7}$$

with

$$Q_1 < Q_2 < \cdots < Q_I < 0, \tag{A8}$$

$$0 < Q_{I+1} < Q_{I+2} < \cdots < Q_N. \tag{A9}$$

Thus, we have

$$\sum_i s_i Q_i(t) = 0, \tag{A10}$$

$$\sum_i s_i^2 Q_i < S_I \sum_i S_i Q_i = 0. \tag{A11}$$

Thus Eq. (A3) can be simplified to

$$\frac{ds(t)}{dt} = M + \frac{(M+2W)s(t)}{(2M+2W)t}, \tag{A12}$$

which yields

$$s(t) = (2M + 2W)t. \tag{A13}$$

Similarly, from Eq. (A3), we have

$$\frac{dQ(t)}{dt} = M^2 + \frac{(M+2W)Q(t)}{(M+W)t} + 2F(t)\sum_i s_i^2 Q_i(t), \tag{A14}$$

which yields

$$\frac{dQ(t)}{dt} < M^2 + \frac{(M+2W)Q(t)}{(M+W)t}, \tag{A15}$$

which results in

$$Q(t) \le Ct^{(M+2W)/(M+W)} \tag{A16}$$

in the case of $t \to \infty$, where C is a constant. So we have

$$Q(t) \ll s^2(t) \quad (t \to \infty). \tag{A17}$$

Then, from Eq. (A3), we obtain

$$\frac{ds_1}{dt} = \frac{(M+2W)s_1}{(2M+2W)t} + F(t)s_1Q_1(t), \tag{A18}$$

which yields

$$\frac{ds_1}{dt} < \frac{(M+2W)s_1}{(2M+2W)t}. \tag{A19}$$

Hence, we have

$$s_1 \leq Ct^{(M+2W)/2(M+W)} \quad (t \to \infty), \tag{A20}$$

which results in

$$\frac{F(t)s_iQ_i(t)}{[(M+2W)s_i/s]} \sim \frac{Q - s_is}{s^2 - Q} \to 0, \tag{A21}$$

when $t \to \infty$. Therefore, the term $F(t)s_iQ_i(t)$ can be safely dropped from Eq. (A3).

[1] R. Pastor-Satorras and A. Vespignani, *Evolution and Structure of the Internet: A Statistical Physics Approach* (Cambridge University Press, Cambridge, England, 2004).
[2] L. A. N. Amaral, A. Scala, M. Barthélemy, and H. E. Stanley, Proc. Natl. Acad. Sci. U.S.A. **97**, 11149 (2000).
[3] R. Albert, H. Jeong, and A.-L. Barabási, Nature (London) **401**, 130 (1999).
[4] S. N. Dorogovtsev and J. F. F. Mendes, *Evolution of Networks: From Biological Nets to the Internet and WWW* (Oxford University Press, Oxford, 2003).
[5] M. E. J. Newman, Phys. Rev. E **64**, 016132 (2001).
[6] A.-L. Barabási, H. Jeong, Z. Néda. E. Ravasz, A. Schubert, and T. Vicsek, Physica A **311**, 590 (2002).
[7] R. Guimera and L. A. N Amaral, Eur. Phys. J. B **38**, 381 (2004).
[8] A. Barrat, M. Barthélemy, R. Pastor-Satorras, and A. Vespignani, Proc. Natl. Acad. Sci. U.S.A. **101**, 3747 (2004).
[9] W. Li and X. Cai, Phys. Rev. E **69**, 046106 (2004).
[10] R. Guimera, S. Mossa, A. Turtschi, and L. A. N. Amearal, Proc. Natl. Acad. Sci. U.S.A. **102**, 7794 (2005).
[11] D. J. Watts and S. H. Strogaz, Nature (London) **393**, 440 (1998).
[12] A.-L. Barabási and R. Albert, Science **286**, 509 (1999).
[13] R. Albert and A.-L. Barabási, Rev. Mod. Phys. **74**, 47 (2002).
[14] S. N. Dorogovtsev and J. F. F. Mendes, Adv. Phys. **51**, 1079 (2002).
[15] D. J. Watts, Annu. Rev. Sociol. **30**, 243 (2004).
[16] M. E. J. Newman, SIAM Rev. **45**, 167 (2003).
[17] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang, Phys. Rep. **424**, 175 (2006).
[18] S. L. Pimm, *Food Webs*, 2nd ed. (University of Chicago Press, Chicago, 2002).
[19] A. E. Krause, K. A. Frank, D. M. Mason, R. E. Ulanowicz, and W. W. Taylor, Nature (London) **426**, 282 (2003).
[20] K.-I. Goh, B. Kahng, and D. Kim, Phys. Rev. E **72**, 017103 (2005).
[21] A. Barrat, M. Barthélemy, and A. Vespignani, Phys. Rev. Lett. **92**, 228701 (2004); Phys. Rev. E **70**, 066149 (2004).
[22] T. Antal and P. L. Krapivsky, Phys. Rev. E **71**, 026103 (2005).
[23] A. Barrat and R. Pastor-Satorras, Phys. Rev. E **71**, 036127 (2005).
[24] Z.-X. Wu, X.-J. Xu, and Y.-H. Wang, Phys. Rev. E **71**, 066124 (2005).
[25] K.-I. Goh, B. Kahng, and D. Kim, Phys. Rev. E **72**, 017103 (2005).
[26] W.-X. Wang, B. Hu, T. Zhou, B.-H. Wang, and Y.-B. Xie, Phys. Rev. E **72**, 046140 (2005).
[27] P. Van Mieghem and S. M. Magdalena, Phys. Rev. E **72**, 056138 (2005).
[28] T. Kalisky, S. Sreenivasan, L. A. Braunstein, S. V. Buldyrev, S. Havlin, and H. E. Stanley, Phys. Rev. E **73**, 025103(R) (2006).
[29] Z. Pan, X. Li, and X. Wang, Phys. Rev. E **73**, 056109 (2006).
[30] G. Mukherjee and S. S. Manna, Phys. Rev. E **74**, 036111 (2006).
[31] S. Maslov and K. Sneppen, Science **296**, 910 (2002).
[32] R. Pastor-Satorras, A. Vázquez, and A. Vespignani, Phys. Rev. Lett. **87**, 258701 (2001).
[33] M. E. J. Newman, Phys. Rev. Lett. **89**, 208701 (2002); Phys. Rev. E **67**, 026126 (2003).
[34] A virtual round table on ten leading questions for network research can be found in Eur. Phys. J. B **38**, 143 (2004), special issue on applications of networks, edited by G. Caldarelli, A. Erzan, and A. Vespignani.
[35] W.-X. Wang, B. Hu, B.-H. Wang, and G. Yan, Phys. Rev. E **73**, 016133 (2006).
[36] S. N. Dorogovtsev and J. F. F. Mendes, Europhys. Lett. **52**, 33 (2000).
[37] S. N. Dorogovtsev, J. F. F. Mendes, and A. N. Samukhin, Phys. Rev. Lett. **85**, 4633 (2000).
[38] W.-X. Wang, B.-H. Wang, B. Hu, G. Yan, and Q. Ou, Phys. Rev. Lett. **94**, 188702 (2005).
[39] E. Ravasz and A.-L. Barabási, Phys. Rev. E **67**, 026112 (2003).
[40] S. Maslov and K. Sneppen, Science **296**, 910 (2001).
[41] S. H. Yook, H. Jeong, A.-L. Barabasi, and Y. Tu, Phys. Rev. Lett. **86**, 5835 (2001).
[42] D. Zheng, S. Trimper, B. Zheng, and P. M. Hui, Phys. Rev. E **67**, 040102(R) (2003).