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Accelerating consensus of self-driven swarm via adaptive speed

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ABSTRACT

In recent years, the well-developed Vicsek model has attracted more and more attention. Unfortunately, in-depth research on its convergence speed is not yet completed. In this paper, we investigate some key factors governing the convergence speed of the Vicsek model with the assistance of extensive numerical simulations. A significant phenomenon surfaces that the convergence time scales obeys a power law with $r^2 \ln N$, with r and N being the horizon radius and the number of particles, respectively. To further accelerate the convergence procedure, we propose a kind of improved Vicsek model with self-driven particles governed by variational speeds, which can remarkably shorten the convergence time of the standard Vicsek model.

1. Introduction

In nature, collective motions of abundant organisms universally exist in biological flocks/swarms/schools, ranging from the behavior of groups of ants [1], colonies of bacteria [2] and clusters of cells [3] in the microcosmic scale, to migration of flocks of birds and schools of fish [4] in the macroscopical scale. These different forms of collective behavior root in the different kinds of interactions among group members, and hence the investigation on the inter-individual interactions among self-driven swarms has attracted more and more attention among physicists, biologists, as well as social and systems scientists. Its value is two-fold: (i) examine the nature of such collective behaviors among bio-groups; (ii) extract some generic rules from those natural systems, and apply them in other relevant industrial application realms, such as sensor network data fusion, load balancing, swarms/flocks, unmanned air vehicles (UAVs), attitude alignment of satellite clusters, congestion control of communication networks, multi-agent formation control, and so on [5–8].

Inspired by biological collective motion, Vicsek et al. [9] described each individual in the collective motion as a self-driven particle moving with a constant speed and adjusting the direction according to the average direction over their neighborhood. By this means, the moving direction of the whole group will be synchronized through finite steps [9–12]. Thereafter, many modified models of self-driven swarms are also proposed, of which one of the most representative and realistic models is developed by Couzin et al. [13,14] which incorporates the effects of the repulsion, alignment and attraction, and thereby provides the different forming mechanisms of three typical bio-group formations, i.e. swarm, torus and flock.

In a word, the simple mechanism-driven Vicsek model is of great academic significance [15], which has become a well-accepted platform to investigate the consensus of bio-groups. Most previous works, however, focus on the analysis of steady

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state, and pay little attention on the time consumed to achieve the steady state, which should be more intensively explored due to its importance of studying the transient state of the bio-group dynamics. More precisely, the convergence speed analysis can yield a criterion of evaluation of a consensus strategy. Moreover, further investigation on the relationship between some key parameters and the convergence speed can shed some light into the nature of bio-groups' collective dynamics. As a consequence, in this paper, we will extract the key factors determining the convergence time, based on which design a mechanism driving the multi-agent system to the synchronized status as quickly as possible. First, we will discuss the relationship between the convergence time and the particle density as well as the horizon radius of individuals. Afterwards, considering the fact that the speed of each individual in a natural biological group or an industrial multi-robot system should be variational in a certain range rather than a fixed value, we relax the constant speed assumption in the Vicsek model into a variational speed one, upon which a novel adaptive communication protocol is designed. Extensive numerical simulations show that the convergence procedure governed by this protocol is remarkably accelerated compared with the counterpart of the classical Vicsek model.

2. Convergence time in the Vicsek model

A group of N particles are considered to move in an $L \times L$ square with periodic boundary conditions. In the Vicsek model, the particles are moving in an identical constant speed but different directions, with initial positions and moving directions of the particles randomly distributed in the square and the angular interval $[-\pi, \pi)$, respectively. At each time step, the direction of each particle is determined by averaging the moving directions of all the particles (including itself) within the circle centered by itself with horizon radius r. Mathematically speaking, the position of the ith particle is updated according to

$$\vec{x}_i(t + \delta t) = \vec{x}_i(t) + \vec{v}_i(t)\delta t, \tag{1}$$

with the corresponding discrete-time presentation writing:

$$\vec{x}_i(t + 1) = \vec{x}_i(t) + \vec{v}_i(t). \tag{2}$$

And its direction is updated by

$$\theta_i(t + 1) = \langle\theta_i(t)\rangle_r + \Delta\theta_i, \tag{3}$$

where $\Delta\theta_i$ denotes the noise, and $\langle\theta_i(t)\rangle_r$ represents the average direction of all the particles within the horizon radius r, including itself. $\Delta\theta_i$ is a variable randomly distributed in the interval $[-\eta/2, \eta/2]$. Obviously, $\langle\theta_i(t)\rangle_r$ is given by

$$\tan[\langle\theta_i(t)\rangle_r] = \frac{\langle v_i \sin\theta_i(t)\rangle_r}{\langle v_i \cos\theta_i(t)\rangle_r}. \tag{4}$$

Moreover, in order to measure the synchronization performance the group, an index [6] is introduced as

$$\phi = \left|\frac{\sum_{i=1}^{N} \vec{v}_i}{\sum_{i=1}^{N} |\vec{v}_i|}\right|, \quad 0 \le \phi \le 1. \tag{5}$$

A larger value of ϕ implies a better consensus performance, especially when $\phi = 1$, all particles move towards the same direction. For the high density/low noise case, all the particles will definitely approach the consensus status, namely reach the identical moving direction after finite time steps [10].

Due to the limitations of the horizon radius, each particle can only communicate with its neighboring particles within the radius and change its direction according to this local information. Different horizon radii and particle densities will give rise to diverse convergence behaviors and times, which will be intensively investigated later. Without loss of generality, the area is fixed, and hence the particle density can be directly represented by the number of particles, N.

The simulation results about the convergence time T based on the noise-free Vicsek model ($\eta = 0$) are shown in Fig. 1, where $T = \min_{\phi_t \ge 0.95} T$ represents the minimal time steps number t reaching the value 'tube' of $\phi_t \ge 0.95$. As long as the threshold ϕ_t is large enough (close to 1), the variance of its specific value will have little influence on the qualitative results shown in this paper. It is observed from Fig. 1 that the convergence time decreases with increasing radius and density, which lies in the following two aspects: (i) Given a fixed particle density, the larger the horizon radius, the less steps taken to reach synchronization, since larger r can constitute more valuable information than smaller r, and thereby drive each particle to the final convergence direction more quickly. (ii) Given a fixed horizon radius, when the number of particles increases, although the percentage of particles which communicate with a given particle does not increase as well, a higher density is also helpful to reduce the convergence time, because the particle is making more integral adjustments at each time step with the assistance of the increased particles inside its horizon (those particles possess different moving directions from different areas a time step before, and thereby yield motion information at the latest time step, afterwards passing such moving direction information to different areas at the following time step).

As shown in Fig. 2, it can be deduced from extensive numerical simulations that the convergence time T follows a power law with $r^2 \ln N$, i.e.,

$$T \sim (r^2 \ln N)^{-1.29}. \tag{6}$$

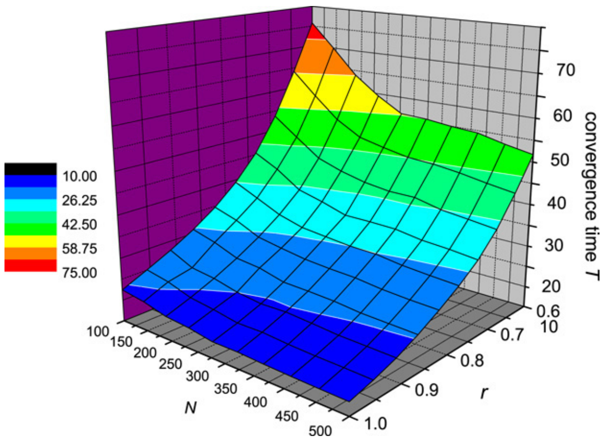

Fig. 1. (Color online) The role of horizon radius r and the number of particles N on the convergence time T. In the simulation, all the particles move in a square shaped plane of linear size $L = 5$, with a constant speed as $v = 0.05$. Control parameters r and N, respectively, vary from 0.5 to 1 and from 100 to 500. The convergence time is obtained from the average over 500 independent runs.

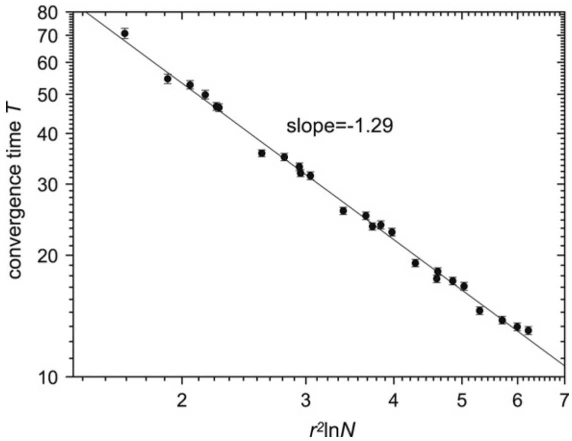

Fig. 2. Convergence time T as a function of $r^2 \ln N$. The data points can be well fitted linearly in double logarithmic coordinates, with slope and error bars marked in the plot. Each data point is an average over 500 independent runs.

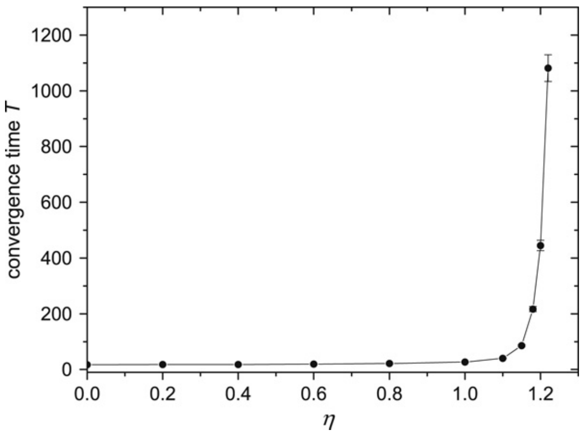

Fig. 3. Convergence time T VS the noise η, here $r = 1.0$ and $N = 100$.

In the case with noise, namely $\eta > 0$, the convergence time will increase with the noise magnitude. As shown in Fig. 3, the convergence time increases slightly if $\eta < 1.1$, conversely, when $\eta > 1.1$, the convergence time increases dramatically, especially when $\eta > 1.2$, the synchronization performance ϕ cannot reach the synchronization threshold of 0.95, which implies that the collective motion cannot achieve a consensus with a high amount of noise. This result is consistent with

Fig. 2 of Ref. [9]. Moreover, we have also implemented simulation for different values of (r, N, ϕ_t), the sharp increasing of T (as the increasing of ϕ) can also be observed in such cases, and the generality of the results of Fig. 3 is thus verified.

3. Fast convergence collective motion with variational speed

In the following context, it is assumed that the the number of particles is fixed, or the generation of new particles or the annihilation of old particles will never occur. In addition, it is not economic to accelerate the consensus process merely by expanding the horizon radius, which implies higher requirement on both technology and the hardware cost. Considering the variability of particle speed [16,17], we propose a new consensus strategy which can effectively shorten the convergence time compared with the classical Vicsek model.

With a limited horizon radius, a particle could make a decision solely based on the local information it receives. In a completely chaotic case, although each particle updates its direction by averaging its neighboring particles's directions, such average direction may be far different from the final synchronized one. Therefore, it makes more sense to act with a comparatively conservative strategy, say, taking a relatively lower velocity to avoid the unnecessary deviation of its current position. Here the unnecessary change means that, if a particle changes its position hastily just in order to communicate with another group of particles under such chaotic circumstance, it is clear that such impatient behavior is not beneficial. By contrast, if all the particles choose a conservative strategy adopting a lower moving velocity, then each particle will have sufficient time to communicate with its neighbors, and hence has the potential to yield a faster convergence. Only when a certain moving direction is dominant among a particle's neighborhood, it is rational to align the particles's direction to that one and with a relatively higher velocity since in that case it is unnecessary for this particle to continue hesitating. In brief, a particle's velocity should be somehow be determined by its local consensus performance.

In order to measure the performance of local synchronization, a local index ϕ_i of particle i is introduced by

$$\phi_i = \left| \frac{\sum_{j=1}^{N_i} \vec{v}_{ij}}{\sum_{j=1}^{N_i} \left|\vec{v}_{ij}\right|} \right|, \quad 0 \le \phi_i \le 1, \tag{7}$$

where N_i is the number of particles within the horizon radius of the ith particle (including itself). The larger value of $\phi_i(t)$, the better consensus performance is achieved by the neighbors of the ith particle. Especially, when $\phi_i(t) = 1$, all i's neighbors move towards a same direction.

In order to compare with the classical Vicsek model in terms of the convergence time, we assume that the magnitude of speed varies from 0 to 0.05. According to the discussion mentioned above, this consensus strategy with variational speed should satisfy the following two conditions:

(a) When all the neighbors of particle i arrive at an ordered direction of $\phi_i(t) = 1$, one has $v_i(t + 1) = 0.05$;

(b) When $\phi_i(t) = 0$, namely the motions of the particles in i's neighborhood is completely disordered, one has $v_i(t + 1) = 0$.

Accordingly, we set the speed of the ith particle as:

$$v_i(t + 1) = v_{\max} e^{\beta[\phi_i(t)-1]}, \tag{8}$$

with $v_{\max} = 0.05$, and β representing a tuning parameter. When $\beta = 0$ the protocol degenerates to the classical Vicsek model, while for $\beta > 0$, a particle will move faster in a more synchronized local circumstance. Actually, in the present protocol, speed not only determines the particles' positions in the next time step, but also transmits the information of the local synchronization performance inside the group. The moving direction of the ith particle is also updated by Eq. (4). Note that, when ϕ approaches to 1, ϕ_i gets close to 1 as well. Therefore, it follows from Eq. (8) that the speeds of all particles at this time are approximating 0.05, which automatically ensures the consensus on the absolute speed.

In sum, the present protocol with adaptive speed can be described as follows:

(1) Determine the initial position and speed of every particle i;
(2) Evaluate the local synchronization performance of each particle, and determine its next direction and speed according to Eqs. (4) and (8);
(3) Calculate the current synchronization performance ϕ of all particles;
(4) Repeat (2) and (3) until ϕ approaches 1.

In the numerical simulations, we first consider the situation without noise ($\eta = 0$), and still assume that all particles move in a square with $L = 5$. The relation between the convergence time T and the horizon radius r and the number of particles N with different β is shown in Fig. 4. It is observed that in this new protocol the convergence time T also obeys a power function with $r^2 \ln N$, the same rule as illustrated in Section 2. Especially, when β is not large enough ($\beta < 5$), the exponents almost posses the same value of 1.30. More significantly, it is found that, compared with the classical Vicsek model, the convergence time is effectively shortened, which demonstrates the virtue of this modified model in accelerating the convergence process.

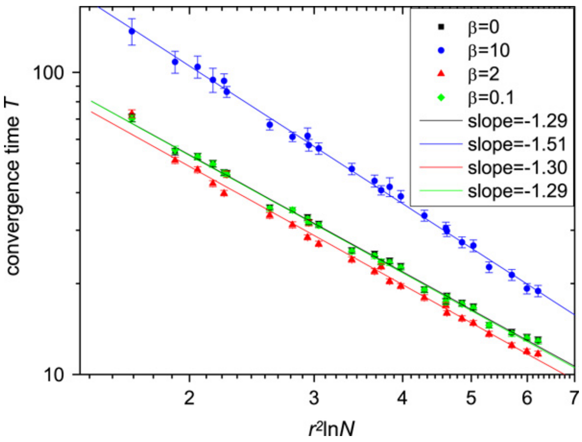

Fig. 4. (Color online) Convergence time T as a function of $r^2 \ln N$ with different β. The data points are well fitted linearly in a double logarithmic coordinates, with slopes and error bars marked in the plot. Control parameters r and N, respectively, vary from 0.5 to 1 and from 100 to 300. Each convergence time is an average over 500 independent runs.

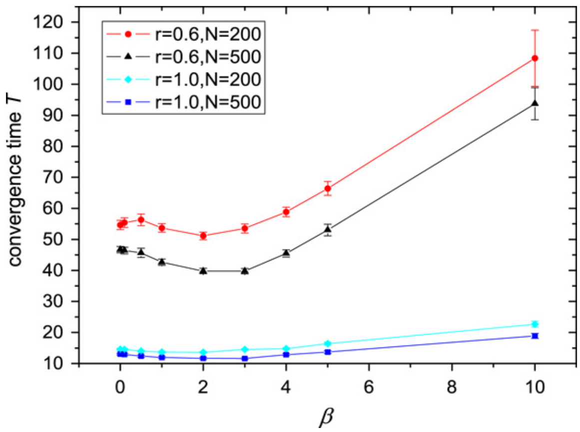

Fig. 5. (Color online) Convergence time *VS*. β with different r and N.

To further explore this point, we fix r and N, and try to find out the optimal β corresponding to the shortest convergence time. In Fig. 5, one can find that the optimal β is always 2 regardless of different values of r and N. On the other hand, the convergence time is sharply shortened for small values of r.

In Fig. 6, one can observe that when β is small (e.g. $\beta = 0.1$), no matter how the current speed is distributed, the speeds of most particles in the next time will approach to the upper limit 0.05, resulting in roughly the same performance as the classical Vicsek model; conversely, when β is too large (e.g. $\beta = 10$), the speeds of particles will tend to 0, and hence even when the neighbors of a given particle are relatively ordered, it still keeps rest and therefore cannot intercommunicate with others, which yields an extremely long convergence time. For a proper value of β (e.g. $\beta = 2$), when a particle has a highly synchronized local performance, it will have a large amount of confidence on its current direction, and hence leave the area with a high speed and using the direction information to influence the others. Otherwise, it will cast doubt on its current direction, and wander with a relatively slow speed in order to avoid misguiding others with its unconfirmed direction.

4. Conclusions

The collective behavior of intelligent agents is not only a wide-spread phenomena in nature, but also an important problem requiring in-depth investigation in industrial engineering. However, most of the previous studies concentrated on the depiction and modeling of the swarm itself, and the systematical analyses about the convergence time were rarely reported. In this paper, we have studied the relationship between the convergence time and the particle density as well as the horizon radius of the well-accepted Vicsek model, and found that the convergence time T obeys a power function of $r^2 \ln N$. Furthermore, we have designed a novel motion protocol with variational moving speed, which also transmits some information about its situation of local synchronization. With such protocol, the convergence procedure is remarkably accelerated compared with the classical Vicsek model. Note that, the advantages of the resent protocol will worsen along with an increasing magnitude of the noise.

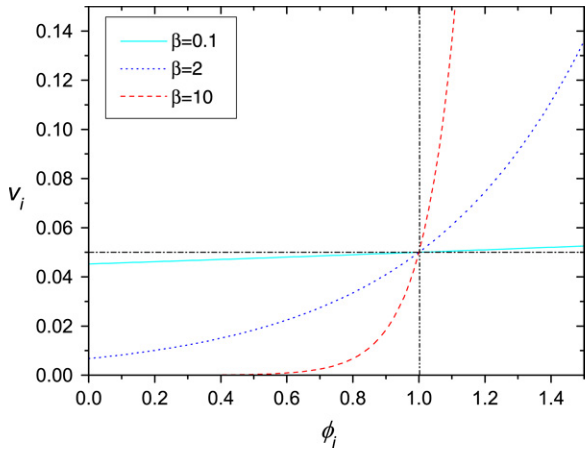

Fig. 6. (Color online) Function $v_i(\phi_i) = 0.05e^{\beta[\phi_i - 1]}$ for $\beta = 0.1$, 2 and 10.

Still worth-mentioning is that the important role convergence time plays is three-fold. First, the study of convergence time is beneficial to the understanding of some natural phenomena. For example, the swarm of fish and birds can always hold consensus under the varying of the direction and speed of the leader's motion, which can not be well explained by traditional models with local interactions since such systems require very long convergence time suffered by the change of leader's direction. A recent work [18,19] highlighted the predictive intelligence embedded in organisms, by which collectively moving agents achieve the consensus within a very short time period. Therefore, it is expected that the predictive intelligence actually plays a crucial role in a real biological swarm. Second, the convergence time is also somehow important for economic systems. For example, some equilibria that can be predicted theoretically can not be observed in real life, since the convergence time is too long. For instance, a recent paper provided a systematic investigation of convergence time to Nash equilibria [20]. Finally, the convergence time can be considered as a measure of efficiency, especially for the design of engineering systems [21].

This work provides a starting point aimed at accelerating the consensus procedure of flocks/swarms by using adaptive speed protocol, and we hope that it will open new avenues in this fascinating direction.

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