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A weighted network model for interpersonal relationship evolution

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Abstract

A simple model is proposed to mimic and study the evolution of interpersonal relationships in a student class. The small social group is simply assumed as an undirected and weighted graph, in which students are represented by vertices, and the depth of favor or disfavor between them are denoted by the corresponding edge weight. In our model, we find that the first impression between people has a crucial influence on the final status of student relations (i.e., the final distribution of edge weights). The system displays a phase transition in the final hostility proportion depending on the initial amity possibility. We can further define the strength of vertices to describe the individual popularity, which exhibits nonlinear evolution. Meanwhile, various nonrandom perturbations to the initial system have been investigated, and simulation results are in accord with common real-life observations.

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Keywords: Weighted network; Interpersonal relationship; Nonlinear evolution

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1. Introduction

Recently, physicists have displayed a lot of interest in social phenomena that exhibit complex behaviors and nonlinear dynamics. A social network is a set of people with some pattern of contacts or interactions between them [1,2]. The patterns of friendships between individuals [3,4], business relationships between corporations [5,6], and intermarriages between families [7] are all examples of networks that have been studied in the past. Hidden behind such complex phenomena, however, are many factors hard to control, including human nature, social environment, social distance and also fate. Social psychologists have long noticed that the first impression between two people is often the seed of their future relationship, not only emerged in romantic movies. This effect seems to be more distinct in campus life, where the influence of social distance is not remarkable. Pupils of a class serve as a typical example that exhibits relatively simple friendships. See the studies of friendship networks of school children by Rapoport [4]. According to empirical observation, students are more likely to get along with their friends in daily activities, such as dinner, discussion, entertainment, etc. Often, their ties are strengthened through frequent contacts. Assumably, people with common friends or common "enemies" are prone to unite; likewise, the "enemy" of one's friends or the friend of one's "enemies" may be very difficult to associate with him. Similar human relations and social environments should be an effective catalyzer for friendships. On the other hand, as a result of restricted social scope and psychological sensitivity, the encounters between "enemies" in a class could also become quite frequent. You cannot avoid your foes in such a small world. It might be an unfriendly eyesight, a provocative action or an unexpected quarrel, as commonplace in daily life. Arguably, in the deepest of one's heart always stay those he hates or he loves, while people without often contacts could be easier to fade from memory.

In this paper, we shall propose a simple model to study interpersonal relationships within a class. The small social world is assumed as an undirected and weighted graph, where students are represented by vertices, and the depth of favor or disfavor between them are denoted by the weight of corresponding edge. It is worth remarking that the model is not necessarily restricted to pupil relations, but can be also applied to other cases, such as relationships in a club or a team. There are many uncontrollable but important ingredients affecting interpersonal relations; for example, temper, gender, ethnicity, and wealth status. We do not attempt to include many types of factors in this paper. Instead, we shall only concentrate on the following empirical principle: similar human relations between people will promote friendship, while opposite interpersonal relationships may lead to hostility. We are most interested in what phenomena can be observed by focusing on this plausible point while idealizing other conditions. This attempt may risk losing important information from microscopic interacting mechanism and diversity of agents. But it can hopefully serve as a simple and basic framework for further theoretical studies. Our paper is organized as follows. In Section 2, we describe the model on some simplified assumptions. Next, through computer simulations, we discuss the relations between initial configurations and the final distribution properties of the system. In Section 4, the nonrandom perturbations to the system and its corresponding nonlinear evolving behaviors are particularly discussed. Finally, we conclude in Section 5 with review of previous experimental researches and outlook of possible applications.

2. The Model

The model system consists of N individuals. Since the size of a class is not too large, it is reasonable to assume that each student has chances to contact all his classmates. For clarity, we introduce a generalization of the $N \times N$ adjacency matrix to describe the interpersonal relationships of the small social group. The matrix elements ω_{ij} represent the weight of edge e_{ij}, where i, $j = 1, 2, \ldots, N$. We postulate that the value of ω_{ij} is discrete and can be negative for the case of disfavor relations. If most elements of the matrix are positive, the system is called harmonious; otherwise, it contains considerable hostility. As an original model, we add one assumption for simplicity that each contact can only alter the regarding edge weight by ± 1 at most. In other words, love or hatred is not formed in one day (or individuals will not fall in "love" at first sight). This condition makes the personal interactions moderate and could be interpreted by the fact that true friends or enemies should be selected by time. For undirected and weighted graphs, $\omega_{ii} = 0$ and $\omega_{ij} = \omega_{ji}$. In the following, we will only discuss the case of symmetric weights (asymmetric case, though universal in real life, will be discussed in future). Since the ith row of the matrix records the information of interpersonal relationships of student i, we will use it to define the individual popularity and the interpersonal similarity. The latter point is of a basic assumption in our model and will be reviewed soon. At the beginning of evolution, it is natural to assume that many initial weights have nonzero values, due to the first impression between any two. For convenience, we assign value 1 with probability p, and -1 with probability $1 - p$ to the nondiagonal elements, i.e., the seed of this model is given. Here, p is called *the initial amity possibility*, which generally describes the social inclination to be friendly in first association. The matrix symmetry, as a requirement of the model, must be satisfied automatically all the time. One may see that the initial configuration of the system is also moderate (all the initial weights are ± 1 at most). It hence allows us to employ nonrandom and acute perturbations to the system. The evolution of our model is defined on the following weights' dynamics:

(i) First, suppose student i has been randomly selected from the class. Then, he takes the initiative to contact student j with a certain probability. A natural idea is to set this possibility as

$$P_{i \to j} = \frac{|\omega_{ij}|}{\sum_{j=1}^{N} |\omega_{ij}|} \, . \tag{1}$$

This equation means that student i is more inclined to contact those students he/she clearly likes (or dislikes). However, it may lead to the absurd case that individual

j with $\omega_{ij} = 0$ would not be noticed by i, nor would i by j. Therefore, any edge with zero weight remains invariant throughout the evolution; that is, strange people would be forever strange. To avoid this unrealistic scenario, we let i choose j with possibility

$$W_{i\to j} = \frac{|\omega_{ij}| + 1}{\sum_{j=1}^{N}(|\omega_{ij}| + 1)} \tag{2}$$

and obviously,

$$W_{i\to j} = W_{j\to i} \,. \tag{3}$$

One could of course change the strategy of "plus one" to that of "plus two" or others. Such changes have been tested on computer, yielding similar results. As an original model, however, we just try to keep it moderate and avoid too factitious means. The problem with zero weights seems inescapable if one intends to model an evolving social group with both friends and foes. To be evolving, all the weights should be permitted to increase or decline. To include both friends and foes, the zero weights must appear through the weight fluctuations, where time plays an irresistible role. Everyone may have the chance to be forgotten temporarily by some of his acquaintances.

In a fresh environment, people will try to get familiar with others and we call it unfamiliar–familiar period. The differences of weights are not significant at present; thus, the contacts between them display no obvious preferences. Once some weight becomes zero during this period, it is still possible to be altered in later contacts. Hence, at an initial stage, the interpersonal relations of the class are not stationary, and the early impressions rising then will play a crucial role in future weight evolution. When friends and enemies (large $|\omega_{ij}|$) have formed in the system, things are quite different: contacts between friends or encounters between enemies now will become more frequent (according to the above equation), and the interpersonal relationships tend to be steady. We can call it the friend–enemy period, in which the emergence of new friends and new enemies is still allowed.

(ii) Now, i and j have been chosen for personal interaction, then ω_{ij} is supposed to be altered with certain possibility:

$$\omega_{ij} \longrightarrow \omega_{ij} \pm 1 \,. \tag{4}$$

We hold that similar human relations and social environments are more likely to promote friendships (when idealizing all the other conditions). So we define the possibility as below to describe the interpersonal relation similarity:

$$\gamma_{ij} = C^{-1} \sum_{k} \omega_{ik} \cdot \omega_{kj} \,, \tag{5}$$

where

$$C = \sqrt{\sum_{k} \omega_{ik}^2} \cdot \sqrt{\sum_{k} \omega_{kj}^2} \,. \tag{6}$$

It is manifest that $\gamma_{ij} = \gamma_{ji}$ and $-1 \leqslant \gamma_{ij} \leqslant 1$. A signed possibility cannot be used directly, but as below:

$$\omega_{ij} \to \omega_{ij} + Sgn(\omega_{ij}) \,, \tag{7}$$

with possibility $|\gamma_{ij}|$.

After the weights have been updated, the process is iterated by randomly selecting a new individual for the next contact, i.e., going back to step (i) until the class disbands.

The mechanism (ii) has simple physical and realistic interpretations. Take Jack and Mike for instance. If they have common friends or common enemies, they are more likely to strengthen their friendship ($\gamma > 0$). Now suppose, they are good fellows at first, yet Mike's pals are all Jack's foes. If Jack goes on associating with Mike, he may be excluded by his own friends and has to confront his foes in Mike's circle. Thus, their social relations have a potential to separate them ($\gamma < 0$), just like an electron–positron system under external electro–magnetic field. In this case, we can say that Jack and Mike equally have distinct social tastes and in the long run, their friendship, if any, is on a harsh test. To better understand this micro dynamics, it maybe helpful to further analyze the case for $N = 3$. Suppose Jack(A), Mike(B) and John(C) interact with each other according to above mechanism. The possible states

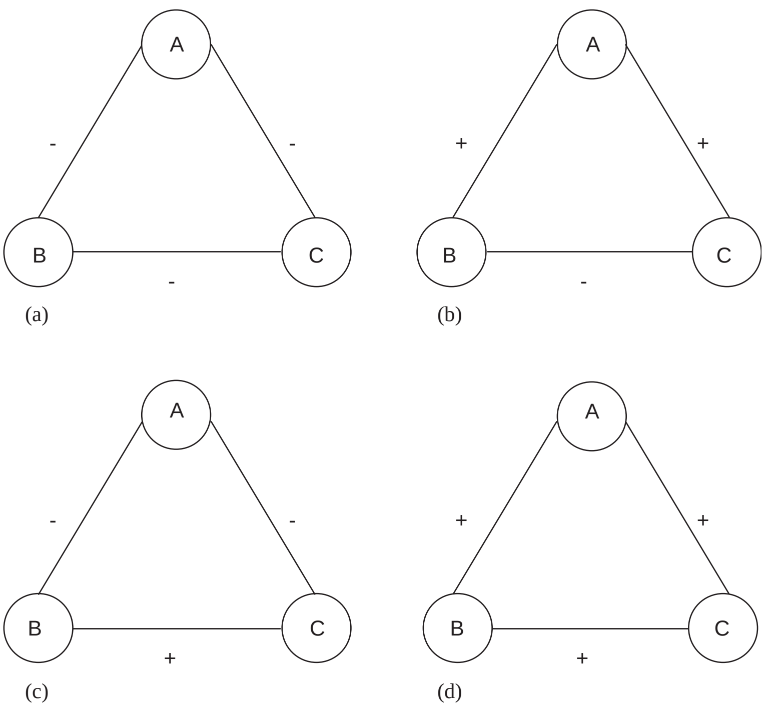

Fig. 1. Possible states of triangle relationship. Positive(+) edge means friendly relation and negative(−) represents hostility.

of this triangle-relation evolution are shown in Fig. 1. Triangle (b) represents the friend–friend–enemy relation, that is, two edges of the triangle have positive(+) weights and another is negative(−). However, the common friend of the antagonistic two will play a conciliatory role in the evolution, and thus the negative weight will be neutralized at some point, given sufficient interacting time. On the basis of similar analysis, triangle (c) and (d) are expectantly the most possible stationary states in the evolution. The asymmetry of the micromechanism will lead to the asymmetric weight distribution at the macro level. This point will be further proved in the next section.

One may notice that the definition of γ_{ij} is equivalent to the inner product of two normalized vectors. Why not use a nonlinear form, such as $\gamma_{ij} \sim \sum_k \omega_{ik}^{\alpha} \cdot \omega_{kj}^{\alpha}$? It may make sense, but α here should be odd to allow $\gamma_{ij} < 0$. Admittedly, there are too many complicated mechanisms that influence human association, which in nature is nonlinear. It is yet impossible to stand on points. For necessary simplicity, we will still use the present form of Eq. (5), which could be very representative at this very point: similar social circle promotes friendship, while opposite interpersonal relationships lead to hostility. We are most interested in how the interactions between social circles can affect interpersonal relations. On the other hand, the present form is quite adaptable for further improvements. Our model is kept as simple as possible, and will demonstrate what could generally happen if its basic idea accepted.

The most commonly used topological information about vertices is their degree and is defined as the number of neighbors. A natural generalization in the case of weighted networks is the strength. To describe the popularity of individual i, define strength s_i as

$$s_i = \sum_{j=1}^{N} \omega_{ij} \,. \tag{8}$$

We could write the evolution equation of strength s_i

$$\frac{\partial s_i}{\partial t} = \sum_{j=1, j \neq i}^{N} \frac{1}{N} W_{i \to j} \gamma_{ij} + \sum_{j=1, j \neq i}^{N} \frac{1}{N} W_{j \to i} \gamma_{ji} \,. \tag{9}$$

The first term on the right-hand side of Eq. (9) describes all the possible alterations to s_i at time step t, contributed by the cases that i is first randomly chosen and then takes initiative in contacting j; likewise, the second term describes all contributions from j contacting i. Given that $W_{i \to j} = W_{j \to i}$ and $\gamma_{ij} = \gamma_{ji}$, one could simplify master Eq. (9)

$$\frac{\partial s_i}{\partial t} = \frac{2}{N} \sum_{j=1, j \neq i}^{N} W_{i \to j} \gamma_{ij} \tag{10}$$

$$= \frac{2}{N} \frac{\sum_{j=1, j \neq i}^{N} (|\omega_{ij}| + 1) \gamma_{ij}}{\sum_{j=1, j \neq i}^{N} (|\omega_{ij}| + 1)} \,. \tag{11}$$

Weight ω has a certain distribution in the final state, so does γ. For convenience, we define

$$\langle\gamma\rangle_i = \frac{\sum_{j=1,j\neq i}^{N}(|\omega_{ij}|+1)\gamma_{ij}}{\sum_{j=1,j\neq i}^{N}(|\omega_{ij}|+1)} \tag{12}$$

and obviously, $|\langle\gamma\rangle_i| < 1$.

3. Initial configurations vs. final distributions

We choose different *initial amity possibility* p to perform simulations. In order to obtain the weight distribution, the range of weight ω is equally divided by M. Then, the range $[\omega_{\min}, \omega_{\max}]$ becomes $[\omega_1, \omega_2)$, $[\omega_2, \omega_3)$, $\ldots$, $[\omega_M, \omega_{M+1}]$, where $\omega_1 = \omega_{\min}$, $\omega_{M+1} = \omega_{\max}$. Define n_{ω_l} as the number of weights in $[\omega_l, \omega_{l+1})$, $l = 1, 2, \ldots, M$; when $l = M$ the interval is $[\omega_M, \omega_{M+1}]$. We will use the same method to describe other properties later. The model generates a broad range of weight distributions ($n_\omega \sim \omega$ diagrams, typically plotted in Figs. 2–7) by just changing parameter p. From $p = 0$ to $p = 0.5$, the $n_\omega \sim \omega$ diagram all behaves a pinnacle; for $p \approx 0.6$, it appears that the power-law distribution with a heavy tail; for $p \approx 0.7$, an exponential decay; and a peak structure emerges as p approaches 1.0. All the observed weight distribution properties are independent from time steps. When $p = 0$, i.e., the initial nondiagonal elements are all -1, the final weight distribution exhibits a symmetric pinnacle near $\omega = 0$. Similar behaviors are found for $p \leqslant 0.5$ (see Figs. 2 and 3). We can conclude that when the initial amity is insufficient, the harmony of the class is

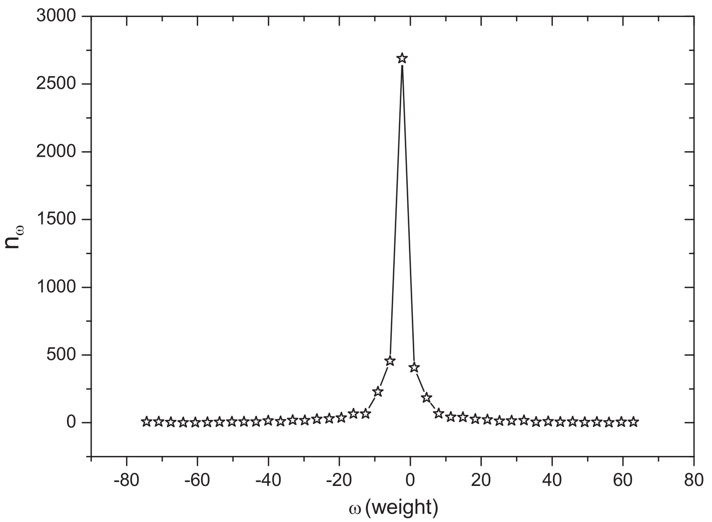

Fig. 2. Weight distribution for $N = 100$, $p = 0$ after 1.0×10^6 time steps.

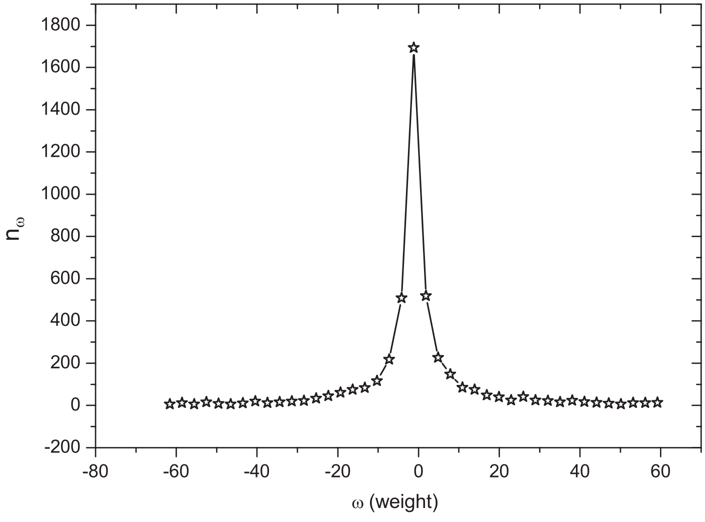

Fig. 3. Weight distribution for $N = 100$, $p = 0.50$ after 1.0×10^6 time steps.

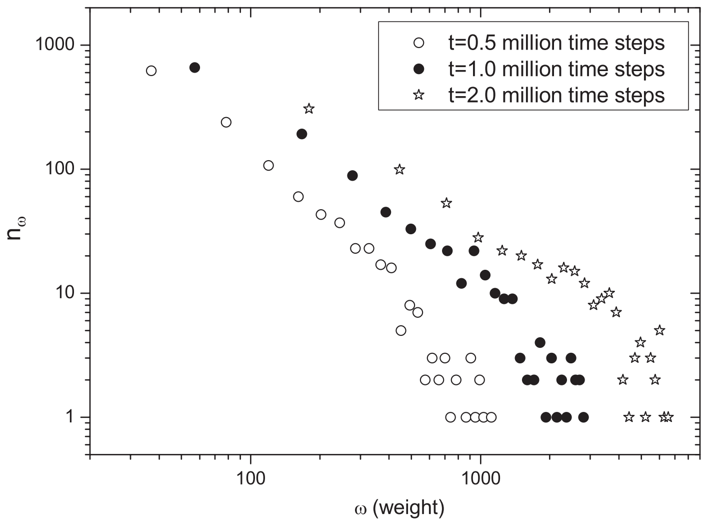

Fig. 4. Weight distribution for $N = 100$, $p = 0.60$ after 0.5×10^6, 1.0×10^6 and 2.0×10^6 time steps.

hard to achieve. The maximum majority of the class are indifferent to others (many elements near zero), and true friends or big foes can rarely "survive" under such environments (few elements far from zero). Positive weight distributions (power-law) for $p \approx 0.6$ are presented in Figs. 4 and 5. The negative weights in the matrix are sparse and are discarded in the log–log plot. Near $p = 0.7$, we further find the

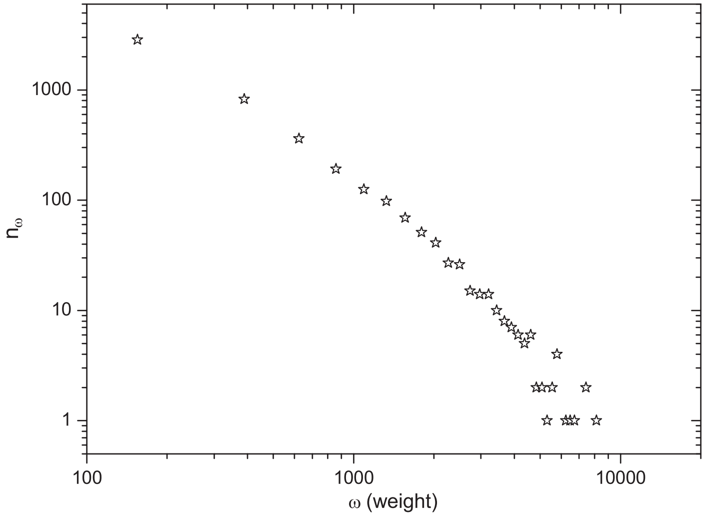

Fig. 5. Weight distribution for $N = 200$, $p = 0.59$ after 9.0×10^6 time steps.

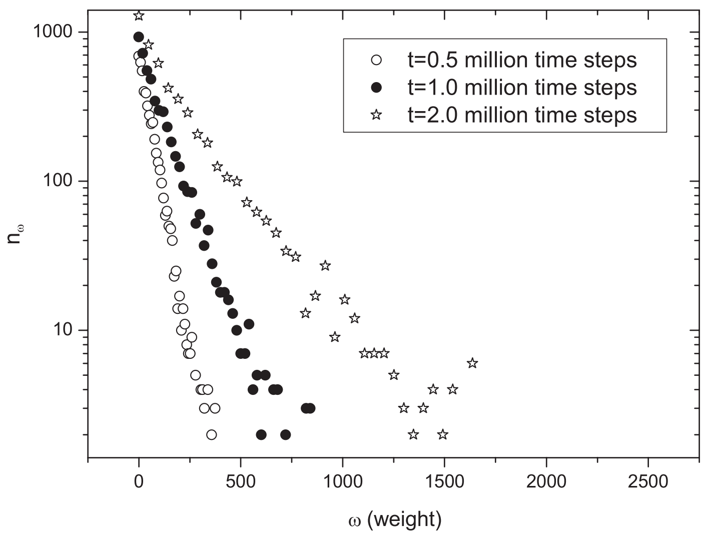

Fig. 6. Weight distribution for $N = 100$, $p = 0.70$ after 0.5×10^6, 1.0×10^6 and 2.0×10^6 time steps.

exponential decay, as shown in semi-log Fig. 6. Clearly, n_ω for large ω increases with time steps, while the negative weights have already disappeared under such circumstances. For $p = 1.0$, the $n_\omega \sim \omega$ diagram is shown in Fig. 7. By increasing the iterated times, its peak and upper limit of ω are both pushed towards the positive. This means the harmony of the system has been boosted up with the

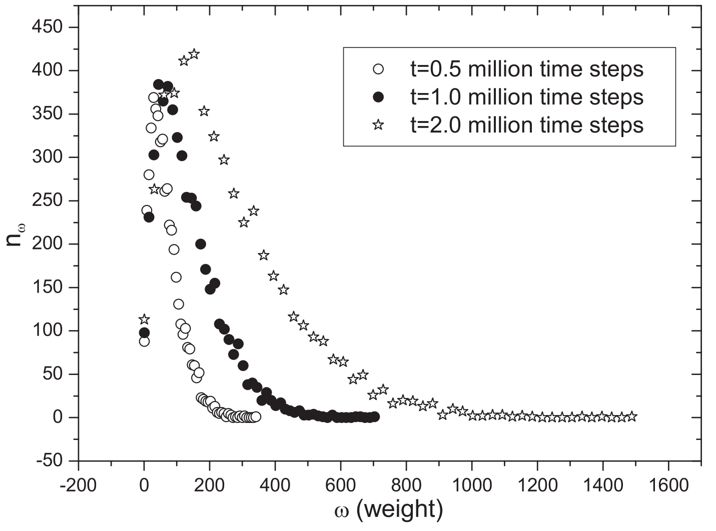

Fig. 7. Weight distribution for $N = 100$, $p = 1.00$ after 0.5×10^6, 1.0×10^6 and 2.0×10^6 time steps.

passage of time. One could check that the weight distribution for $p \geqslant 0.8$ behaves likewise.

By comparing the weight distributions of different p from 0 to 1, one may notice that this system exhibits a potential to become harmonious. This trend has its underlying causes in the micro mechanism (see the analysis of $N = 3$ case). Meanwhile, the properties of the above weight distributions also suggest a critical transition of the system. Define *the hostility proportion h* as

$$\sum_{i,j,\omega_{ij}<0} \omega_{ij} \Big/ \sum\nolimits_{i,j} |\omega_{ij}| \,, \tag{13}$$

which can describe the disharmonious degree of the class. The dependence of h on parameter p is given in Fig. 8, and a phase transition is found near $p_c = 0.6$, where the weight distribution apparently observes power law. Below the critical value, the hostility proportion h is nontrivial, that is, there exists considerable hostility in the interpersonal atmosphere; while above the critical point p_c, the final hostility proportion h is nearly zero, i.e., the class is harmonious. Conflicts and grievances can melt gradually under such an ideally harmonious environment. It may be helpful to further compare the distribution of ω with that of γ for fixed p. As shown in Fig. 9, the γ distribution for $p = 0$ has a pinnacle near $\gamma = 0$ likewise in comparison with Fig. 2. It indicates that when the initial nondiagonal elements are all -1, the evolvement of the system is quite tardy. Many contacts will give no alteration to the weights, or equally, the effective social interactions between people that update their relations rarely occur. At the other extreme, one can tell from Fig. 10 that the γ distribution for $p = 1$ similarly exhibits a peak structure and reaches its apex near $\gamma = 0.67$. Since the weight distribution for $p = 1$ also displays a peak for $\omega > 0$, the

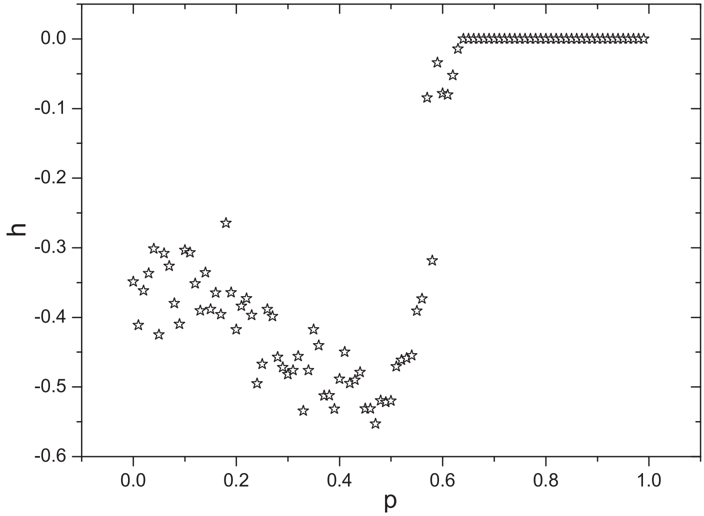

Fig. 8. The dependence of hostility proportion h on the initial amity proportion p, hostility-amity phase transition for $N = 100$ after 1.0×10^6 time steps.

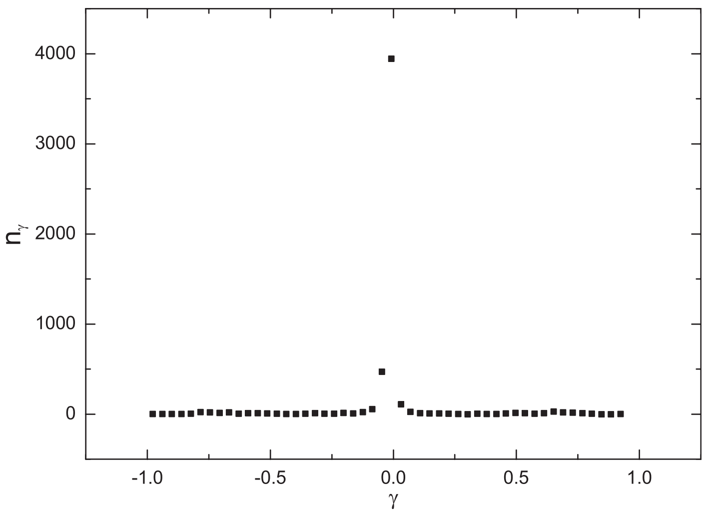

Fig. 9. γ distribution for $p = 0.00$ and $N = 100$ after 1.0×10^6 time steps.

effective contacts between individuals now should be quite frequent. It makes an active group with increasing harmony.

The diversified distribution properties observed by changing the initial amity parameter best illustrate the significance of the interacting mechanism we reiterated

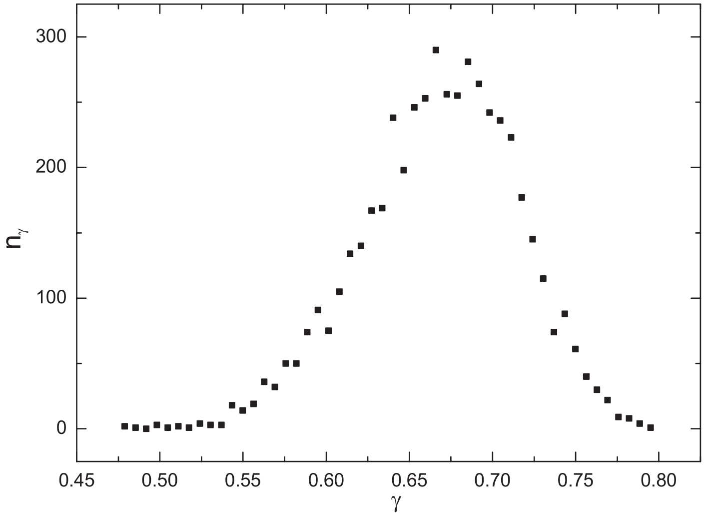

Fig. 10. γ distribution for $p = 1.00$ and $N = 100$ after 1.0×10^6 time steps.

before. Without considering the microscopic diversity of individuals, it can even obtain various macroscopic phenomena, indicating that the mechanism may be quite responsible for the apparent complexity of real life. Also noticeable is that the phase transition in final hostility proportion depending on initial impressions could be a general property for a family of congeneric models. As simulations reveal: above the critical point, all the individuals will evolve to one family of great harmony; while below it, two large antagonistic clusters will emerge in the evolution. For some more simple models (e.g. an unweighted model), we could even analytically verify this property and calculate the critical point. All too often, however, more than two clusters are observed in a real social community. To better understand it, one should consider the individual diversity and related mechanisms. Of course, our model already serves as a good groundwork for this possibility.

4. Perturbations and nonlinear evolution

In the original model, we suppose that individuals at first are not familiar with each other. The initial configuration of the system is determined by the initial amity possibility p which can be interpreted as the first impressions among them. However, it makes sense that two or more individuals have good fellowship or deep grievances previously. Plotted in Fig. 11 is the $\omega_0 \sim t$ diagram, describing the process of an old grievance between two people soon "thawing" in a harmonious group ($p \approx 1.0$). It can be interpreted from two aspects. On the one hand, the new environment will provide them sufficient opportunities to contact other people, so they would not encounter too frequently and aggravate their grievance. On the other hand, they will

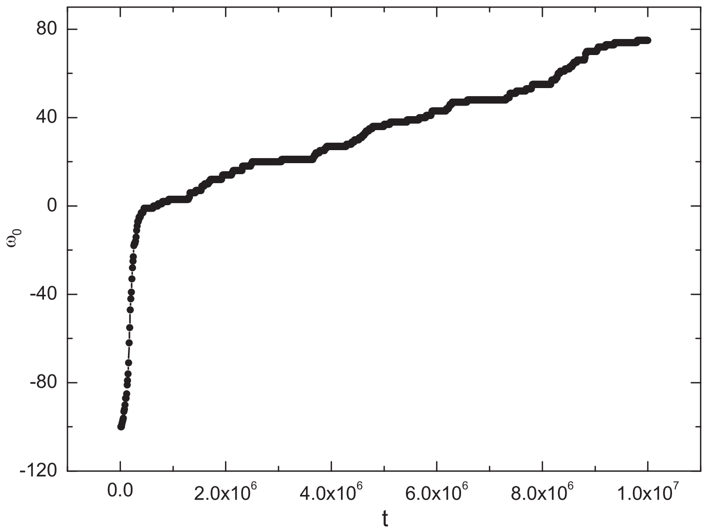

Fig. 11. Weight evolution for $N = 100$ during 1.0×10^7 time steps. Initially, all the weights are set as $+1$ except $\omega_0 = -100$, where ω_0 denotes the designated negative weight (the old grievance).

quickly get along well with other people in the new environment, due to $p \approx 1.0$ and the first impression effects. Therefore, after sufficient contacts the two will have a common circle of friends that play important roles in mitigating their old grievance. Admittedly, this scenario is too ideal, because some grievances in real life could be immitigable and a perfectly harmonious group is hard to find. But from the case one could easily feel that an initial harmonious group has such a significant ability to mitigate the grievances it contains. In the following, we will consider nonrandom perturbations to the moderate system just in such extreme cases as $p = 0$ or 1. Between the two extremes, the phenomena may be more relevant to real life, yet of less clarity to our major conclusions.

An individual could have significant potential to change the interpersonal relationships of a collective. All too often, a social group has its leader, who owns great reverence among his people. He may be an elderly chief, a captain of a football team or a chairman of an association. The leader's men may constantly have conflicts and grievances. We are interested in the role of the leader playing in the development of the group. We assign a large integer to the matrix elements at row i ($\omega_{ij} = \omega_{ji}$ and $\omega_{ii} = 0$) to denote the leader's relations, and set all the other nondiagonal elements as -1, representing the common conflicts of the masses. In Fig. 12, we have taken this integer as 100. Apparently, the leader's strength grows linearly, while the strength of his members increases with a slower velocity. In comparison with the evolution containing no leader, one could conclude that the strengths of the system have been remarkably boosted up by the leader. The leader has changed the possible state of disunity and make his men draw together. The slope of the leader's popularity evolution is 0.03167. In

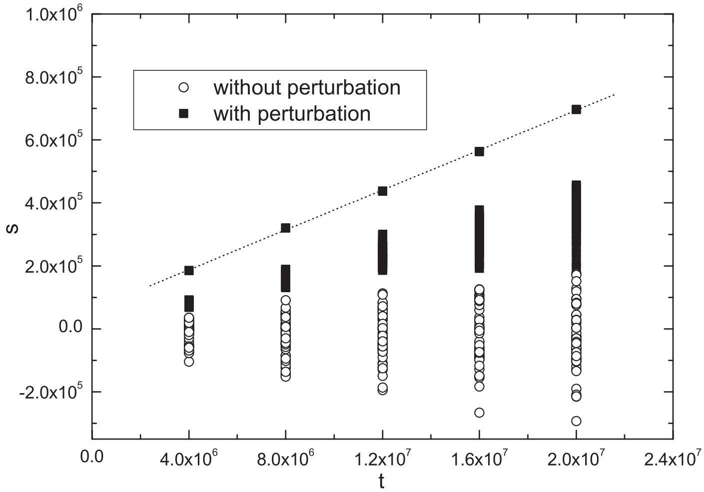

Fig. 12. Strength evolution for $N = 50$ and $p = 0.00$ during 2.0×10^7 time steps. The dotted line (with slope 0.03167) fits the strength evolution of the leader, and black-squares(■) represent the strengths of the others, and circles(○) denote the strength evolution without the leader, under the same conditions. The data are drawn out at intervals of 4.0×10^6 time steps.

Section 2, we have

$$\frac{\partial s_i}{\partial t} = 2\langle\gamma\rangle_i/N\ . \tag{14}$$

For the case containing the leader, if we assume that $\omega_{kj} \ll \omega_{ij}$ for all j and $i \neq k$, then $\gamma_{ij} \approx 1$, and further $\partial s_i/\partial t \approx 2/N = 0.04$, in approximate agreement with the numerical result 0.03167. For the case without the leader, the average velocity of the popularity growth can be estimated too. We suppose $\langle\gamma\rangle_i \approx \langle\gamma\rangle$, and $\langle\gamma\rangle$ is numerically calculated as -0.0024. Then Eq. (12) gives the average velocity of popularity growth -9.60×10^{-5}, which is nearly zero. The master equation (14) can only give an average rate of popularity growth which is not precise, as one may see in Fig. 12.

It is also interesting to study the opposite case, say, of putting a public anti into a united group. For initialization, we assign -100 to the matrix elements at row i ($\omega_{ij} = \omega_{ji} = -100$ and $\omega_{ii} = 0$) to represent the anti's relation state, and set all the other nondiagonal elements as 1 to describe the united group. Obviously, the anti's popularity decreases linearly, while the strength of the public rises at a relatively slow speed, see Fig. 13. Comparing with the case containing no anti, the strength evolution of the group has not been evidently influenced by the anti. However, the anti's incompatibility to the united group is aggravated. Supposing $\langle\gamma\rangle_j \approx \langle\gamma\rangle$ for $j \neq i$ and considering that $\langle\gamma\rangle = 0.4060$ (by numerical calculation), we could estimate the average growth rate of the public strength as $2\langle\gamma\rangle_j/N \approx 2\langle\gamma\rangle/N = 2 \times 0.4060/50 = 0.01624$. This is consistent with the strength evolution of the public, as shown in

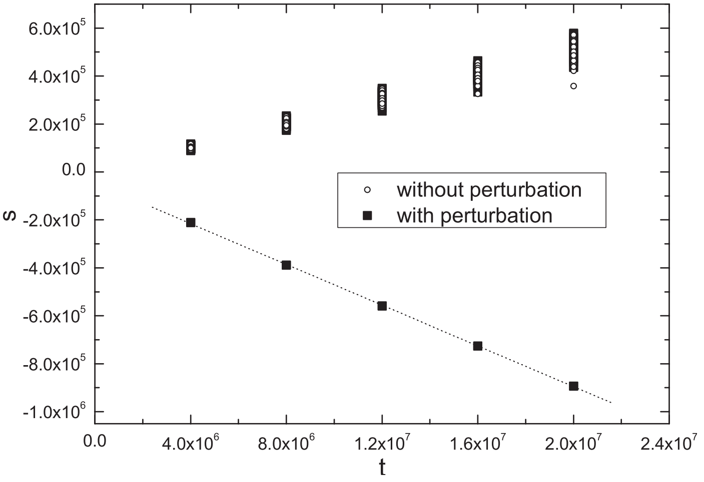

Fig. 13. Strength evolution for $N = 50$ and $p = 1.00$ during 2.0×10^7 time steps. The dotted line (with slope -0.0425) fits the strength evolution of the public anti, and black-squares(■) represent the strengths of the public, and circles(○) denote the strength evolution without the anti, under the same conditions. The data are drawn out at intervals of 4.0×10^6 time steps.

Fig. 13. For the anti, it is reasonable to assume that $|\omega_{kj}| \ll |\omega_{ij}|$ for all j and $i \neq k$, then $\gamma_{ij} \approx -1$ and further $\partial s_i / \partial t = 2/N = -0.04$, in fine agreement with the simulation result -0.0425. In our model, one can see that a completely harmonious group will absolutely exclude its public anti. While for an incompletely harmonious group, one may check that this exclusion is incomplete.

We must stress that the strength evolution is actually nonlinear, as Fig. 14 indicates. Simulations demonstrate that the strength evolution exhibits obvious nonlinear behaviors after sufficient time, no matter what the initial state is. When $p = 1.00$ the strength evolution appears to be a wide linear zone, but finally all individuals will split in the $s \sim t$ diagram and display clearly nonlinear behaviors. For clarity, we would like to discuss the adaptive process of a student joining a class midway, under the initial background $p = 1.00$ (Fig. 15). The old students' popularity grows linearly (since its slope in the log–log scale is 1), even after inserting the new student. The nonlinear strength growth for the new-coming pupil describes his adaptive process in this new environment, which just lasts a brief time, thanks to the harmony of his class. To sum up, the interaction between the individual and collective could significantly affect the interpersonal relationship states. The diversified phenomena generated by the present model under nonrandom perturbations are not inconsistent with the commonsense of daily life. In some aspects, it indicates that our model have well simulated the complexity of the interpersonal relations in a small social group. To better understand the problem, individual diversity and corresponding interaction mechanisms need to be integrated, and combined with possible empirical evidence.

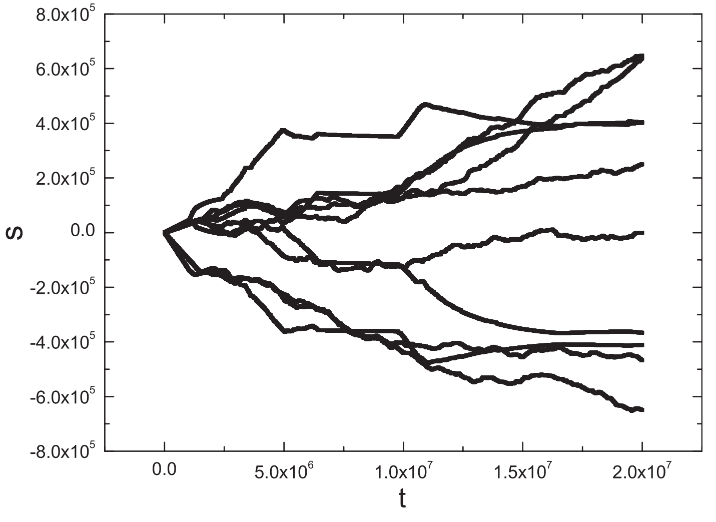

Fig. 14. Strength evolution for $N = 10$ and $p = 0.00$ during 2.0×10^7 time steps.

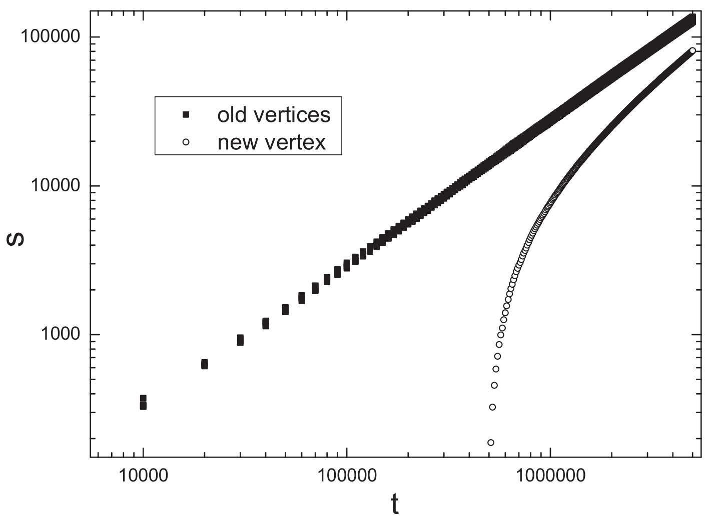

Fig. 15. Strength evolution of a student joining a class ($N = 49$ and $p = 1.00$) midway. The new student joins the class after 4.0×10^5 time steps, and his evolution is denoted by circles(○). The strength evolution of the old students are denoted by black-squares(■). We have extracted five from them as representatives.

5. Review and outlook

Of the academic disciplines, social sciences have the longest history of the substantial quantitative study of real-world networks [8,9]. Of particular note among the early works on the subject are: Jacob Moreno's work in the 1920s and 1930s on friendship patterns within small groups [3]; the so-called "southern women study" of Davis et al. [10], which focused on the social circles of women in an unnamed city in the American south in 1936; the study by Elton Mayo and colleagues of social networks of factory workers in the late 1930s in Chicago [11]; the studies of friendship networks of school children by Rapoport and others [4,12]; and the mathematical models of Anatol Rapoport [13], who was one of the many theorists to stress the significance of the degree distribution in networks of all kinds, not just social networks. In more recent years, studies of business communities [5,14,15] and of patterns of sexual contacts [16–20] have attracted particular attention. However, traditional social network studies often suffer from problems of inaccuracy, subjectivity, and small sample size. With the exception of a few ingenious indirect studies such as Milgrams famous "small-world" experiments, data collection is usually carried out by querying participants directly using questionnaires or interviews. Such methods are labor-intensive and therefore limit the size of the network that can be observed. Survey data are, moreover, influenced by subjective biases on the part of respondents; how one respondent defines a friend for example could be quite different from how another does. Although much effort is put into eliminating possible sources of inconsistency, it is generally accepted that there are large and essentially uncontrolled errors in most of these studies. Because of these problems many researchers have turned to other methods for probing social networks. One source of copious and relatively reliable data is collaboration networks [21,22]; another source of reliable data about personal connections between people is the communication records of certain kinds [23,24]. One could construct a network in which each (directed) edge between two people represented a letter or package sent by mail from one to the other. No study of such a network has been published as far as we are aware, but some similar things have. For example, Ebel et al. have reconstructed the pattern of email communications between five thousand students at Kiel University from logs maintained by email servers. In this network, the vertices represent email addresses and directed edges represent a message passing from one address to another.

We return to our model and relevant discussions. It does not require as large sample size as above for empirical studies. Previous researches had stressed the significance of the degree distribution in large social networks, while the finite size of a class excludes the necessity of measuring the degree properties. Researchers may have to turn to other measures and ingenuous methods. Considering the relative simplicity of student relationships, it is seemingly possible to investigate the (weighted) relations in a certain class. However, to measure the depth of favor or disfavor between people is a question beyond technology. One may expect to see more practical studies on small social weighted networks, but presently we could not find direct empirical data to compare with the theoretical results we obtained. Still, as one can see, our model and related work are of interest and meaning. First, the model is actually built on a weighted network, considering both the depth and development of human relations. Previous theoretical researches just concentrate on topological properties of networks. But a weighted evolving network is more reasonable because the weights of links more reflects the physical significance and processes on the topological network structure. Second, since many social systems display community structures, our model could effectively mimic certain subsystems whose members are interconnected closely as students in a class; in this way, we could further mimic the large social system (which have empirical data for statistical analysis) by properly combining its subgroups on the present basis. Therefore, our model is very extendable in structure. Third, a lot of other ingredients and interaction mechanism could be integrated into the model with little difficulty. It provide a capacious room for possible extensions and variations. One could give each individual various attributes he is interested in; for example, color the students with certain colors, denoting different economic status or races. Finally, the present model, perhaps the simplest, could generate a broad range of phenomena consistent with real world. It offers some general properties produced by one simple mechanism. For instance, the transition from the emergence of two antagonistic clusters (or cliques) to the overall harmony of the system. This phenomenon is not only of interest as it appears, but also means a rudiment of herd behaviors and herd effects due to general interaction mechanism. More cliques are expected to emerge by considering more microscopic diversity and common ground of individuals.

Recently, Alain Barrat, et al. have proposed a general model for the growth of weighted networks [25], considering the effect of the coupling between topology and weights' dynamics. It appears that there is a need for a modelling approach to complex networks that goes beyond the purely topological point. The simple model here is also a step to this point. It could be easily extended to directed graph more close to real world where human relations are often asymmetric. Moreover, generalizing it to complex social, economic and political networks is an interesting and challenging topic too. The relationships between individuals, economic entities or nations are amazingly similar in many aspects. The basic assumptions and concepts here are expected to have applications in other related fields.

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