Networks Emerging from the Competition of Pullulation and Decrepitude *

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In real-world network evolution, the aging effect is universal. We propose a microscopic model for aging networks, which suggests that the activity of a vertex is the result of the competition of two factors: pullulation and decrepitude. By incorporating the pullulation factor into previous models of aging networks, both the global and individual aging effect curves in our model are single peaked, which agrees with the empirical data well. This model can generate networks with scale-free degree distribution, large clustering coefficient and small average distance when the decrepitude intensity is small and the network size not very large. The results of our model show that pullulation may be one of the most important factors affecting the structure and function of aging networks and should not be neglected at all.

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Many social, biological, and communication systems can be properly described as complex networks with vertices representing individuals or organizations and edges mimicking the interactions among them.[1−3] Examples are numerous: these include the Internet,[4] the World Wide Web,[5] social networks of acquaintance or other relations between individuals,[6−8] metabolic networks,[10] and many others.[11−14] In the past few years, with the computerization of data acquisition process and the availability of high computing powers, some novel topological properties have been found, such as small-world effect,[15] scale-free property,[16] nontrivial degree–degree correlation,[4,17] and so forth.

The ubiquity of complex networks inspires scientists to construct a general model. One of the most well-known models is Watts and Strogatz's small-world network (WS network).[15] Another significant model is Barabási and Albert's scale-free network model (BA network),[16] which suggests that two main ingredients of self-organization of a network in a scale-free structure are growth and preferential attachment. Recently, highly active development in the study of modelling networks has revealed many probable underlying evolution mechanisms of networks. However, we should not ask for an all-powerful model which can explain why a freewill real-world network comes into being. It is more meaningful to construct a microscopic model aiming at the particular underlying mechanisms of each kind of network.

A particular mechanism for network evolution is the so-called aging mechanism.[11,18−23] Some empirical studies about citation networks suggest that aging may be one of the most important mechanisms that determine the network topology.[19,20] Figure 1 shows the average number of citations of a paper received in 1998 as a function of the paper publication year.[20] The curve clearly exhibits a single peaked behaviour, which means that neither the youngest nor the eldest individual is the most active one.

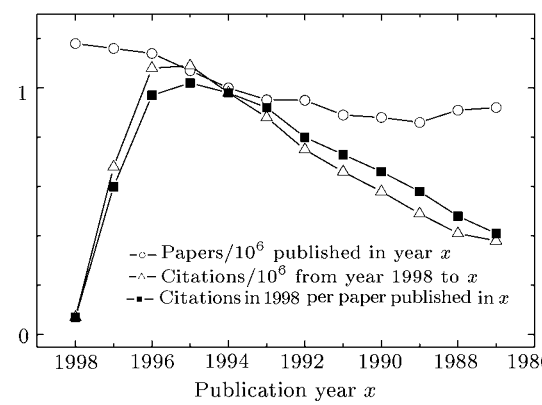

Fig. 1. Data on the network formed by scientific publications and citations. Circles denote the number of papers published in a given year from 1987 to 1998; triangles denote the total number of citations made in papers published in 1998 and referring to papers published in a given year; filled squares denote the average number of citations of a paper, received in 1998 as a function of the paper publication year. The values are obtained to be the ratio between the values of the two curves in the upper panel. The rate of obtaining new citations exhibits a single peaked behaviour versus age.

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As in the empirical studies, it is reasonable that an actor or a scientist loses his vigour gradually, and then his influence on the whole evolution of the network, i.e. the probability of collaborating with new members, will become lower and lower. For the sake of mimicking the case mentioned above, two typical aging models based on a BA model were proposed.[18,21] These kinds of aging models are both defined with a tunable effect of gradual aging: starting with a small number (m_0) of vertices, at every time step, a new vertex is added and attached to $m(\leq m_0)$ existing ones, with the probability proportional to the degree k of the considered vertex, as in the BA model, and to a simple function $f(\tau)$:

$$\Pi_i = \frac{k_i f(\tau_i)}{\sum_j k_j f(\tau_j)}, \qquad (1)$$

where k_i and τ_i is the degree and age of the considered vertex respectively.

Dorogovtsev and Mendes proposed a typical aging function $f(\tau) = \tau^{-\alpha}$ (DM model).[18] We have calculated the probability of the Nth vertex links to the ith vertex at the $(N - m_0)$th time step from the DM model with $\alpha = 0.5$. The result is reported in Fig. 2(a) and named the *global aging effect curve*, for it represents the attaching probability of all vertices from old to young, revealing how the aging affects the network properties as a whole. In Fig. 2(a), one can find that the most active ones are the eldest and youngest ones, which is opposite to the empirical data shown in Fig. 1. The inset shows the attaching probability of a given vertex, the 1000th vertex, versus its age. The corresponding curve is named the *individual aging effect curve* since it reveals the aging effect on one vertex. It is clear that the individual activity decays monotonically, which does not agree with our experience. For example, in the research life of a scientist, he always pullulates relatively fast until the apex in his career, and then declines gradually. It is difficult to imagine that a scientist's fame is very great as soon as he enters a field, and then decreases continuously, as the DM model suggests. Another typical aging function is $f(\tau) = e^{-\beta\tau}$ introduced by Zhu *et al.*[21] We also display the global and individual aging effect curves with $\beta = 0.01$ in Fig. 2(b). Clearly, the exponential aging mechanism built a world including youths only. In order to compare with the non-aging effect case, we report the corresponding results in Fig. 2(c) from the BA model. It is clear that the global age effect curve and the individual one both display monotonous decaying.

Taking one with another, previous studies considered little about the pullulation factor. Therefore, all the previous models deviated from empirical study to some extent. In this Letter, we propose an alternative model for aging networks, which suggests that the activity of a vertex is the result of the competition of the two factors mentioned above: pullulation and decrepitude. To make our description more concrete, we use the language in terms of scientific collaboration networks.[6−8] In scientific collaboration networks, each scientist is represented by a vertex, and the activity of a given vertex can be measured by the number of collaborations of the corresponding scientist within a unit time period. Intuitively, besides preferential attachment, the activity of a vertex depends on two factors as follows:

(i) *Pullulation.* Supposing that a young scientist joins in a new field, his ability and experience of study will both increase, and thus, its activity will improve.

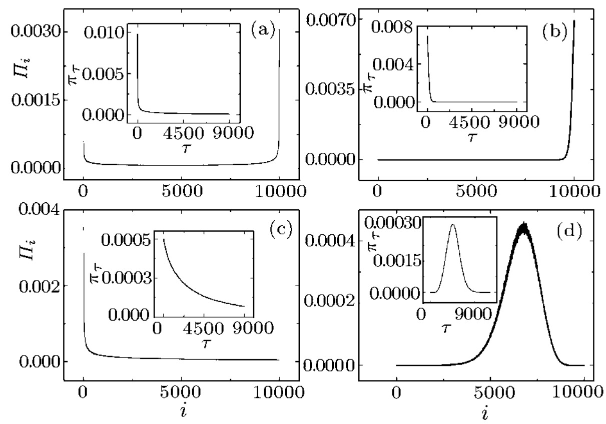

Fig. 2. The global (main plot) and individual (inset) aging effect curves of the DM model with $\alpha = 0.5$ (a), exponential model with $\beta = 0.01$ (b), BA model (c), and the present model with $\alpha = 12$, $\beta = 0.004$ (d). Here the initial $m_0 = 3$ vertices form a complete graph, $m = 3$, and $N = 10000$.

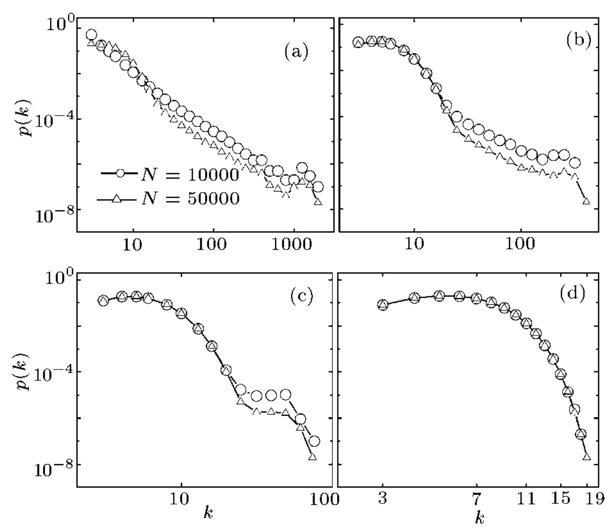

Fig. 3. Log–log plot of the degree distribution of the networks with $\alpha = 3$ and $\beta = 0.0006$ (a), 0.006 (b), 0.06 (c), 0.6 (d), respectively. Here the initial $m_0 = 3$ vertices form a complete graph, and $m = 3$.

(ii) *Decrepitude.* With time elapsing, the scientist's vigour and time spent on study will decrease gradually. Correspondingly, his activity will decrease.

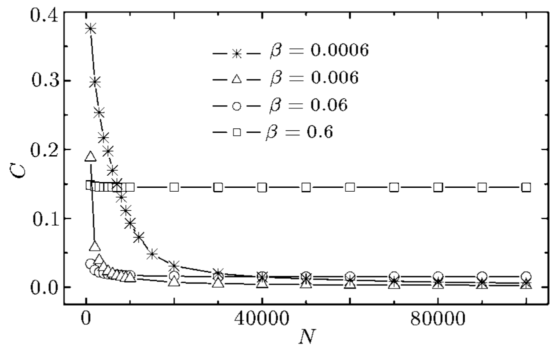

Fig. 4. The clustering coefficient of the networks with $\alpha = 3$. Here the initial $m_0 = 3$ vertices form a complete graph, and $m = 3$.

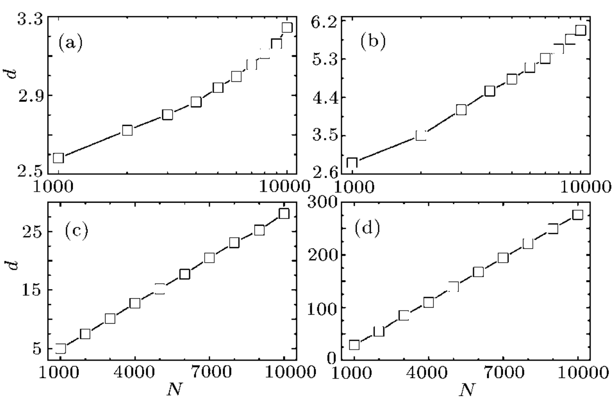

Fig. 5. The average distance of the networks with $\alpha = 3$ and $\beta = 0.0006$ (a), 0.006 (b), 0.06 (c), 0.6 (d), respectively, where (a) and (b) are log-linear. Here the initial $m_0 = 3$ vertices form a complete graph, and $m = 3$.

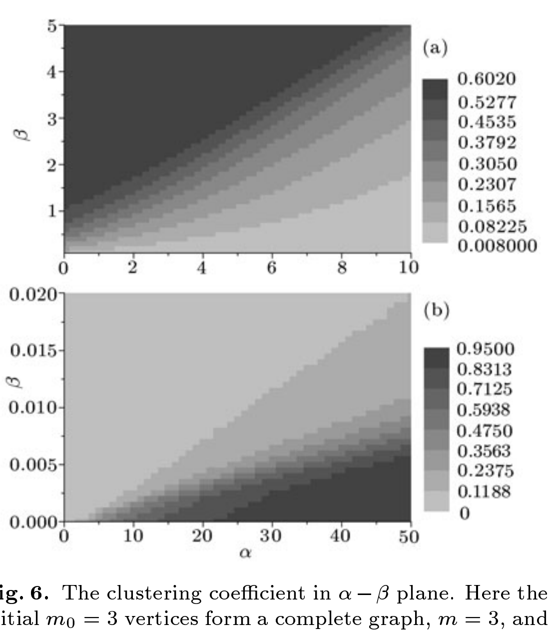

Fig. 6. The clustering coefficient in $\alpha - \beta$ plane. Here the initial $m_0 = 3$ vertices form a complete graph, $m = 3$, and $N = 10000$.

In the present model, the aging function is $f(\tau) = \tau^\alpha e^{-\beta\tau}$, which includes a pullulation factor τ^α and a decrepitude factor $e^{-\beta\tau}$; $f(\tau)$ has a single peak at $\tau = \alpha/\beta$, representing the most active state of a vertex without considering the topological role. Since the exponential function decreases more rapidly than the power one, $f(\tau) \to 0$ (and so $\Pi_i \to 0$) when $\tau \to \infty$. Apparently, the present model will degenerate to the BA model when $\alpha = \beta = 0$, to the DM model when $\beta = 0$, and to the exponential model when $\alpha = 0$, respectively.

In Fig. 2(d), we report a typical experiment on aging effect with $\alpha = 12$, $\beta = 0.004$, both the global and individual aging effect curves are single peaked, which agree with the empirical data well.

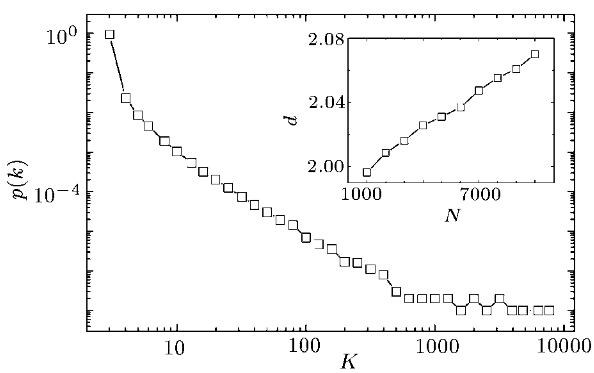

Fig. 7. Log–log plot of the degree distribution of a star-like network with $\alpha = 45$, $\beta = 0.003$, the corresponding clustering coefficient is 0.91. Here the initial $m_0 = 3$ vertices form a complete graph, $m = 3$, and $N = 10000$. This figure shows that almost all the vertices are of degree m ($p(m) \approx 0.95$), and the degree of central vertices is only slightly smaller than the network size N. Of course, such as the star networks, the average distance (inset) is very small.

Figure 3 shows the degree distribution of the networks with $\alpha = 3$ and $\beta = 0.0006, 0.006, 0.06, 0.6$, respectively. Figures 4 and 5 show the clustering coefficient C and the average distance d as a function of the network size N for the same values of α and β as the former. A more microscopic study of clustering coefficient in $\alpha - \beta$ plane is shown in Figs. 6(a) and 6(b). From Fig. 3 we can see that the degree distribution for the fixed values of α and β changes from scale-free to broad-scale and finally becomes single-scale[11] with the increasing network size N. Figure 4 tells us that the clustering coefficient C of the network with the given values of α and β finally approaches a stable value when the network grows larger and larger. Both the transient time will decrease with the decreasing pullulation intensity and the increasing decrepitude intensity. The exponent of the scale-free degree distribution will follow the change of the values of α and β. This is different from the BA model, in which the exponent of the scale-free degree distribution is fixed. However, the exponents of the scale-free degree distribution in real-world networks are different. In this sense, the present model may be closer to reality rather than the BA model. Figure 5 shows that the average distance of the network with small value of β increases logarithmically with the network size N when N is not very large. From Fig. 6 we can see that the clustering coefficient of the network becomes very large when pullulation has an overwhelming majority compared with decrepitude, or by contraries, decrepitude has an overwhelming majority than pullulation. The former two opposite case will result in a star-like network[24] and a chain-like one, respectively. The star-like network is a network whose centre is not one vertex but a small network, as shown in Fig. 7. Figure 8 shows the clustering coefficient C of the star-like network as functions of m_0, m, and K, where K is the degree of a vertex in a nearest-neighbouring couple network formed with initial m_0 vertices. It is well known that the clustering coefficient of the nearest-neighbouring couple network will increase with the increasing K. Figure 8 gives a result that the clustering coefficient C of the star-like network only has a slight change with the increase of m_0 or m, but more seriously depends on the clustering coefficient of the initial network.

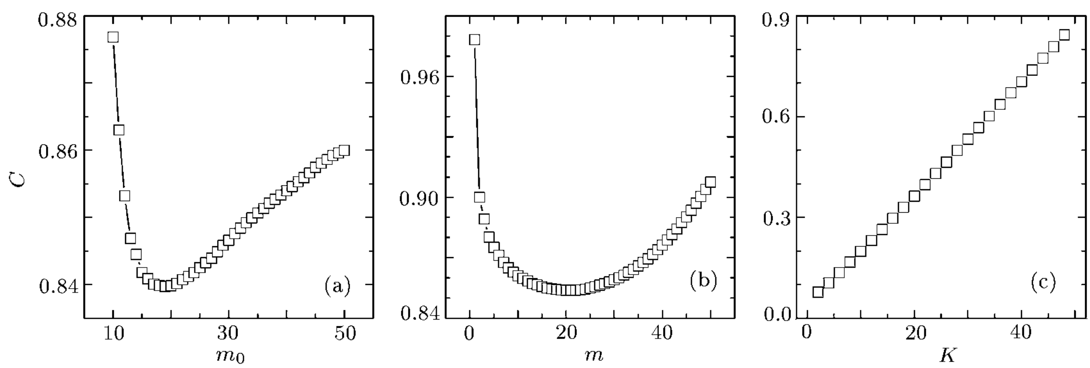

Fig. 8. The clustering coefficient C of the star-like network with $\alpha = 45$, $\beta = 0.003$ as a function of m_0 for $m = 10$ (a), of m for $m_0 = 50$ (b), and of K for $m_0 = 50$ and $m = 10$ (c). In (a) and (b), the initial m_0 vertices are completely connected. In (c), the initial m_0 vertices form a K nearest-neighbouring coupled network. The network size $N = 10000$.

In summary, by incorporating the pullulation factor into previous models of aging networks, both the global and individual aging effect curves in our model are single peaked, which agree with the empirical data well. This model can generate networks with scale-free degree distribution, large clustering coefficient and small average distance when β is small and N not very large. The results of our model show that pullulation may be one of the most important factors affecting the structure and function of aging networks and should not be neglected at all.

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