The effect of investor psychology on the complexity of stock market: An analysis based on cellular automaton model

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A R T I C L E  I N F O

Article history:
Received 22 August 2006
Received in revised form 29 March 2008
Accepted 29 March 2008
Available online 6 April 2008

Keywords:
Cellular automata
Complexity
Hurst exponent
Discrete level
Investor psychology

A B S T R A C T

In this paper, using a developed cellular automaton model of the stock market, variables reflecting fractal and stability properties are introduced to describe complexity in the stock market; the concept of discrete level is defined to characterize market stability. Based on the model, the dependency of market complexity on the investors' imitation degree is investigated. The results show a clear correlation between investors' imitation degree and complexity of the stock market.

1. Introduction

An established method used when exploring complexity in a stock market, is that of an agent-based model titled Artificial Stock Market (ASM), developed by Arthur at Santa Fe Institute (SFI) (Brian, Durlauf, & Lane, 1997). This model, developed since 1989, can be classified as an adaptive nonlinear network. The modeling method is based on interactions among agents by cellular automaton (CA); and has been described in detail by Wolfram (2000) and von Neumann (1951). It is possible for an agent-based model to become a CA model when rational assumptions are made. CA has appeared as an effective tool for simulating complex systems (Cheng, 1998), and has been widely used (Christie & Huang, 1995; Crutchfield & Hanson, 1993; Jerome, 2000; Ying & Wei, 2000).

From the perspective of complex adaptive systems, an economic system includes not only physical elements such as technical experts, all kinds of behaviors, markets, financial companies and factories, but also psychological factors which are not easily apparent but interact with physical elements. In fact, the psychological factors sometimes impact on the macro-economy behavior and cause crisis in finance markets (SFI, 2000). Hence, to understand investors' psychology as a key factor in promoting crises in stock markets has become a focus in contemporary research of economics. The SFI (2000) research has shown that stock markets could collapse in some cases. Results show GARCH behavior, which does not exist in equilibrium theory; that is, the turn-down and continued aberrance will appear as continue upswing when the deviation from the expected course ends. However, those results were obtained by qualitative analysis of the emergence curve; and, moreover, there was no quantitative index to describe the characteristics of stock market behaviors.

Models for herd behavior have been developed, to understand the group behavior in financial markets (Chang & Cheng, 2002; Christie & Huang, 1995; Li, Tan, & Fang, 2002). A paper by Li discussing the behavior of Chinese investors in securities, argues that the usual investment prejudice psychology taken on by Chinese investors, includes determinately loss abhorring, policy depending, imitation, and so on (Li et al., 2002; Li, Wang, & Fu, 2001). All these investment prejudices interact with each other in the market, and hence lead to excess investors’ behavior; and, thereby, speed up the vibration of the security market. In this sense, all the above investment prejudices are examples of negative investment psychology (Li et al., 2001).

In this paper, we consider principally herd behavior as the investment psychology, and use imitation probability to measure this type of behavior. Subsequently, we have defined and quantified these behaviors, and so characterize market behavior. Finally through simulations the correlation between imitation probability
and the global behavior in the market is investigated. The global behavior is defined by numbers of agents who are adopting buying, holding or selling behavior.

2. Model

2.1. CA based modeling method for complex systems

This investigation is based on a cellular-automata-based model for the simulation of investment behavior in a stock market (CASM) (Ying, Wei, & Fan, 2001). The CA based modeling method applied to complex systems, follows the approach of system dynamic modeling. Its underlying principles are described as follows:

1. Identifying the research objects as investors distributed within the stock market and their interactive behaviors.
2. Formulating the computer based simulation model for the object. By defining five concepts (cellular automaton, space of cellular automaton, neighborhood structures, state of CA and evolution rules), we formulate a CASM model, and thus develop a computer program.
3. Running the computer program, simulation experiments via the model can be done. Through different input parameters and test times, as well as evolution numbers, we experiment with varying iterations; this is the original characteristic of the CASM model.
4. Testing the validity of the model. The CASM models a virtual-computerized-stock-market, as does the ASM model developed by SFI (2000). This kind of model has validity only when it fits with real stock markets. The key factors for the CASM model should, therefore, be considered.
5. Explaining the policy implications and providing recommendations to policy makers. In this paper, we investigate the global behavior of the virtual stock market. We investigate what are the relations among financial administration departments, investors and the industrial companies, and what are their situations regarding the stability and collapse of stock markets, and which of these should decision-makers take into account.

2.2. CA model for stock market (CASM)

The model framework in this paper is based on the work developed by Wei, Ying, Fan, and Wang (2003). Its basic assumptions are as follows:

- A cellular automaton represents a stock market. Every site in the cellular automaton represents an investor agent in the stock market.
- The state space of the cellular automaton is a space consists of agents; state variable $S_{ij}(t)$ denotes the investment behavior of the agent at the site $(i,j)$ at time $t$, which can choose the following three values: $S_b$ (represents buying), $S_h$ (represents holding), or $S_s$ (represents selling), that is,
  
  $S_{ij}(t) \in \{S_b, S_h, S_s\}$  

- Evolution rules: Generally, the state change of a given site in a time step will be determined by the states of itself, its neighbors in the previous time step and the control variables, which can be formulated as follows:
  
  $S_{ij}(t+1) = F(S_{ij}(t), S_{ij}(t); G)$  

where

$$
S_{ij}(t) = 
\begin{pmatrix}
S_{ij-1,j}(t), & S_{ij-1,j}(t), & S_{ij-1,j-1}(t), \\
S_{ij+1,j}(t), & S_{ij+1,j}(t), & S_{ij+1,j-1}(t), \\
S_{ij+1,j}(t), & S_{ij-1,j}(t), & S_{ij+1,j-1}(t)
\end{pmatrix}
$$

which represents the states of the neighbor sites (in Moore neighbor) (Wei et al., 2003) for the site $(ij)$, $G$ is the vector of control variables; $F$ is the evolution rules of the cellular automaton.

Under the common rules, it is assumed that cellular states in the stock market at the next time step can only be affected by the investment behavior of neighbors, their own investment preference and macro factors which are represented by $R$ in the model. Since the investment preference can not be easily changed, cellular states, i.e. the investment behavior of an agent is mainly determined by the behavior of most neighbors adopted at the previous time step, besides macro factors also significantly affect the states of agents.

The investment preference is defined as an investor’s imitation behavior. Let $P_{ij}$ denoted the imitation probability of the agent in site $(i,j)$, meaning that its state will change with probability of $P_{ij}$, from its current state to the neighbor’s state, which is the behavior of most neighbors adopted. $P_{ij} \in [0,1]$. When $P_{ij} \in (0,0.5)$ the investment preference is anti-imitation, with its strength depending on how $P_{ij}$ is close to zero; when $P_{ij} \in (0.5,1]$, the investment preference is imitation, the strength of this imitation behavior depends on how $P_{ij}$ is close to 1. When $P_{ij} = 0.5$, the investment preference is non-imitation. In this study, for simplicity, we consider a special situation that the investment preference is the same for all investors, that is, $P_{ij} = P$ for all $i, j$. In a real market, investors’ behavior often appear imitation more or less, so in most cases $P > 0.5$, which is consistent with the ‘majority’ principle. But in this study more possibilities are considered, so $P \in [0,1]$.

With a buying or selling coefficient, the macro factor $M_f \in [0,1]$ increase or decrease the probability of investors’ buying, holding or selling intent. When macro information is positive, the buying probabilities of investors will increase; and when macro information is negative, selling probabilities of investors will increase. $M_f$ is an adjusting coefficient of the transfer probability caused by the macro information. The specific transfer probabilities, in which an agent transfer its behavior from current state to another state, are listed in Table 1.

All values in Table 1 should be limited in the interval $[0,1]$. During the evolution process, values that are less than zero are limited as zero and those larger than 1 are limited as 1. At the same time, the sum of the values of three transfer probabilities for each agent in the cellular automata should be 1. In case the sum is not 1, the evolution program will automatically adjust the transfer probabilities by which the behavior of the current agent is altered to the behavior adopted by most investors.

In this paper, we assume the stock market is a non-constraint market, thus we investigate the impact of investment psychology on the global behavior of the stock market and their corresponding correlations.

2.3. Variables for scale of complexity in stock markets

In this paper, the complexity in stock markets is considered as two aspects: the fractal characteristic and stability. Kinds of time series, such as transaction price, transaction volume, investors’ fortune and so on, could be produced from the CASM model. The fractal characteristics could be tested via the Hurst exponent ($H$) and average cycle length ($L$) obtained from the Re-Scaled Range Analysis (R/S) of specific time series; the stability characteristic could be tested via the discrete level ($W$).
2.3.1. Hurst exponent (H)

To study time series, Hurst (1951) developed a new statistical method: the Re-Scaled Range Analysis (R/S analysis). Then Mandelbrot (1972) and Feder (1998), Sugihara and May (1990), Korvin (1992) introduced R/S analysis of time series as fractal analysis. The fractal properties of time series can also be analyzed by means of R/S method.

Consider a given time series \( \{X_t\} \) with \( t = 1, 2, \ldots, T \), which is produced from CASM model. The average of \( X_t \) over \( N(2 < N < T) \) time steps will be,

\[
< X > _N = \frac{1}{N} \sum_{i=1}^{N} X_i
\]

(3)

The departure from the average over the N-step-horizon is given by

\[
X(i, N) = \sum_{i=1}^{N} |X_i - < X > _N|
\]

(4)

Two key variables are computed from the above time series on average and departure. The first is the standard deviation defined as:

\[
S(N) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (X_i - < X > _N)^2}
\]

(5)

And the second is the so-called range of the time series, given by the difference between the maximum and the minimum over the period 1 to N:

\[
R(N) = \max_{1 \leq i \leq N} X(i, N) - \min_{1 \leq i \leq N} X(i, N)
\]

(6)

Using these variables, the re-scaled range \( F(N) \) is defined as:

\[
F(N) = \frac{R(N)}{S(N)}
\]

(7)

In this way, we have a measurement which scales within the range by taking the standard deviation as the unit of measurement.

Ordinary Brownian motion corresponds to the case \( F(N) \propto N^{1/2} \) where the values of the time series are un-correlated with each other. In terms of forecasting, the best prediction is the last measured value. Therefore, it is crucial to inspect the data, to identify whether such a transition occurs in the In–ln plot, before measuring \( H \). Then, the regression can be drawn through the range of data, to show any evidence of a long memory process (Peters, 1994).

2.3.2. Average cycle length (L)

The Hurst exponent is usually used as a measure of complexity of the trajectory of a cure with a fractal dimension \( D \) where \( D = 2 - H \). Hence a smaller \( H \) means a more complex system. Fractal time series have long, but finite memories. When the related time series of stock markets is studied, similar characteristics could be found, i.e. stock time series are characterized by long but finite memories. Thus the lengths of these memory cycles might be estimated. From R/S analysis of related time series of the stock market, another important variable, average cycle length, could be derived. Nonlinear dynamics theory tells that the observations over the average cycle length will lose its memory for initial information. That is, there is a point in any nonlinear system after where the memory of initial information is lost. This point corresponds to the end of natural period of the system. Therefore, it is crucial to inspect the data, to identify whether such a transition occurs in the ln–ln plot, before measuring \( H \).

3. Simulations and discussion

In order to have sufficient investors (agents) in the simulation space and efficient running speed of the computer simulation, an
adequate size of grid space for the CASM model is needed. Here, a $50 \times 50$ grid space is adopted.

The macro factor in the standard CASM is set to zero since this paper mainly focuses on the relationship between market complexity and investors’ psychology.

In a real market, investors’ psychology varies largely. Thus random imitation probabilities are designed to test market complexity in case 1. On the other extreme, an unvaried imitation probability is designed in case 2 to test how the market complexity is correlated with imitation probability. These two cases will be carefully discussed in this section.

3.1. Case 1: Imitation probability distributing randomly

In this case, the imitation probability of each agent is equally selected from $[0,1]$, so the imitation probability of all agents follows the uniform distribution in $[0,1]$.

3.1.1. The fractal properties in the stock market

In order to test the fractal properties, we perform five experiments involving 3000 steps in each; and the initial state of each agent is randomly equally distributed in buying, holding and selling.

The simulation results show that the evolution space is ultimately at a ‘clutter’ distribution state with buying, holding and selling. The curve of the volumes of all behaviors in the market is full of behavior information regarding the stock market. $H$ and $L$ are estimated for each of experiments. The results show that the behavior volumes are randomly distributed. Fig. 1 is a ln–ln plot on the time series of buying behavior ($C_b$) based on the first experiment. We could identify from the figure that at the point $\ln(N) = 3.36$, there is an abrupt change and it corresponds to $L = 29$.

The calculation of $H$ involves the minimum time steps for observing and minimum number of samples for regression. Suppose the minimum time steps for observing and minimum number of samples for regression is 10, we calculate $H$ corresponding to the observations less than 20, 21, till 1500. The results are displayed in Fig. 2(a). The curve shows that $H$ quickly reaches maximum value and then decrease to 0.5, where the time series loses all memory. Fig. 2(b), which is the partial enlargement of Fig. 2(a), shows that the length of average cycle is $L = 29$, which corresponds to $H = 0.83$.

The $H$ and $L$ of five experiments are listed in Table 2. It is shown that $H$ and $L$ only have small changes among five experiments, where $H$ is from 0.81 to 0.86, and $L$ is from 21 to 29. Theoretically, the Hurst exponent and average cycle length are properties for the whole system, thus they have no direct relationship with initial states. They will have the same value even if the experiments begin from different initial states. These five experiments start from different imitation probabilities coming from the same distribution. It means that they have different initial states. The averages of five experiments are taken as the estimates of $H$ and $L$, which is shown in Table 2, where $H_b = 0.84, L_b = 24$ (for buying state); $H_h = 0.84, L_h = 23$ (for holding state); $H_s = 0.83, L_s = 23$ (for selling state).

The Hurst exponent of the $C$ curve indicates that the evolution of a stock market with randomly distributed imitation probability follows bias ‘walk’; its emergency is ‘trend walk’. In efficient market hypothesis, every investor makes decision independently. The curve of the volume of every behavior in the market follows standard random ‘walk’. However, the result of the experiments is the reverse. A possible explanation for this phenomenon is that investor’s decisions depend on not only the basic analysis and technical analysis, but also perceivable preference and imitation psychology. Traditional analysis takes greater care regarding the information about the word, text, data, table and figure, and takes less care about the imitation psychology. It indicates the stock market has ‘emotion’ when $H$ is greater than 0.5.

The average cycle length of the evolution of stock market is one of important parameters for the predication of economic cycle. If a national economic cycle could be predicated, it would be helpful for investors to understand the market better. Actually the implications of the average cycle length should be understood and explained deeper in future work.

3.1.2. The stability of stock market

In order to test the stability of the market, we perform four experiments with different initial states, that is, the symmetrically initial state, the landscape orientation initial state, the portrait initial state and the random initial state.

Fig. 3 shows that different initial states in the evolving space of the stock market could reach a stable state after 100 evolving steps. The average cycle length of the series of volumes for three types of behaviors in the market is taken as the samples to calculate the expected value and range of expected values corresponding to the states of buying, holding and selling. The results are shown in Table 3.

The discrete level ($W$) corresponding to the above four experiments are 1.46%, 0.14%, 2.39% and 1.49%. If we take 10% as the threshold of discrete level to identify the stability of a market, the final states are deemed stable, although they start from different initial states. We argue that the stability of the market is due to the different behaviors of investors, and that their imitation probability follows uniform distribution in $[0,1]$. Also, the imitation probability varies from zero to one, implies investors’ psychology vary from non-imitation to complete imitation. The volumes of behaviors in the market, among the investors who have different imitation behaviors, are counteracting. Therefore, the stock market with randomly distributed imitation probability is a relatively...
stable market. Fig. 4 shows the volume of behaviors in the market with symmetrically initial state, whilst the other three cases are similar to Fig. 4.

3.2. Case 2: Unvaried imitation probability

In this case, imitation probability of each agent is exactly the same in the whole evolving space; and the initial states of investor behavior in the market is randomly distributed (which is similar to Fig. 3(d)). The share and volume of the investment behaviors in the stock market are {0.32, 0.34, 0.34} and {781, 858, 861}, respectively.

### 3.2.1. The fractal properties in stock market

In this case, five experiments are designed corresponding to imitation probability 1, 0.75, 0.5, 0.25 and 0. Apparently, the emergency of the market in five experiments are different. Table 4 shows the fractal property of the stock market with different imitation probabilities. When the imitation probability equals one, the Hurst exponent of the curve of investment behavior volume in the stock market approximates to one. When the imitation probability equals 0.75, the average cycle length of the investment behavior volume in the stock market is quite large, which means this kind of market has a longer memory on trading information. When the imitation probability equals 0.5, the investment behavior volume of the stock market is similar to the situation with randomly distributed imitation probabilities. The imitation probability changes from large to small as it corresponds to H changes from large to small. But the average cycle length begins to decrease after it increases. So when imitation probability is at a suitable point during (0.5,1), the maximum average cycle length of the evolving market could be obtained.

To investigate the relationship between imitation probability, \( P \), and Hurst exponent of investment behavior, \( H(H_b, H_h \text{ or } H_s) \), more experiments corresponding to 21 values of \( P \) are conducted. A regression through 21 pairs value, the relationship between \( H \) and \( P \) is shown as follows:

### Table 2

<table>
<thead>
<tr>
<th>Items</th>
<th>Buying series</th>
<th>Holding series</th>
<th>Selling series</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( H_b )</td>
<td>( t_b )</td>
<td>( H_b )</td>
</tr>
<tr>
<td>1</td>
<td>0.83</td>
<td>29</td>
<td>0.83</td>
</tr>
<tr>
<td>2</td>
<td>0.83</td>
<td>22</td>
<td>0.83</td>
</tr>
<tr>
<td>3</td>
<td>0.84</td>
<td>24</td>
<td>0.83</td>
</tr>
<tr>
<td>4</td>
<td>0.85</td>
<td>21</td>
<td>0.84</td>
</tr>
<tr>
<td>5</td>
<td>0.86</td>
<td>25</td>
<td>0.86</td>
</tr>
<tr>
<td>Average</td>
<td>0.84</td>
<td>24</td>
<td>0.84</td>
</tr>
</tbody>
</table>
The regression shows that Hurst exponent varies upon imitation probability the same way among three behaviors; the complexity level of investment behavior is proportional to imitation probability. For holding case, the minimum and maximum of the Hurst exponents are 0.48 and 0.99, respectively. When $H_b$ equals to 0.5, $P$ equals to 0.04. That is, when $P = 0.04$, the series of volume of investment behavior is approximately a random walk. Anti-imitation behavior will let the market walk randomly.

3.2.2. The stability of the stock market

The last volume of Table 4 shows the discrete level ($W$), corresponding to $P$ equals of 1, 0.75, 0.5, 0.25 and 0. It indicates that the decreasing of $W$ corresponds to the decreasing of $P$. That is, the smaller imitation probability is, the greater stability the stock market has. Fig. 5 shows the relation between $W$ and $P$.

4. Conclusions

Following the above discussions and analyses on investors’ behavior based on experiments in a simulated stock market, we draw the following conclusions.

(1) When the market is relatively mature, the investment psychology of investors varies and less affects each other, i.e. their imitation degree is not the same. In this situation, the imitation probability of investors could be regarded as randomly distributed. But the experiments clearly show the
fractal characteristics in the market, which might due to the interactions among investment decision making. This indicates that the stock market may not be efficient but follows a bias random walk.

(2) When the herd behavior in the market is in evidence, investors will imitate others. This means investors may conduct imitation or anti-imitation. The experiments show that the market’s stability is highly related to the imitation degree of investors. Greater imitation probability is, less stability of the stock market is. When imitation degree is close to 0, i.e. the investors’ behavior is the opposite of his neighbors’ behavior, the market appears near “walk randomly”. A greater imitation probability corresponds to stronger imitation mentality, that is, investors will be affected more by others’ behavior. Thus, it forms a connected network, which will cause greater fluctuation in the market.

(3) The intensity of trend and the stability in the stock market are related to the psychology of investors. The increasing of intensity of trend corresponds to increasing imitation probability, and the decreasing of a market’s stability corresponds to the increase of imitation probability.

In sum, through the simulation of investment psychology of investors and its emergency in the stock market, it is shown that the investment psychology of investors play an important role in the emergency of the stock market (as we now know what actually happens in the real market). There exists a high correlation between investment psychology and the market behaviors; and this happens in the real market. There exists a high correlation between investment psychology and the market behaviors; and this causes complexity in the stock market. The macro-behavior of the stock market has fractal property and stability. The significance of fractal and diversification of stability are apparent in the complexity of the stock market.

Acknowledgements

The support of the National Natural Science Foundation of China under Grant Nos. 70371064, 70425001, 70573104 and 70733005, is greatly acknowledged. Authors thank Professor Dessouky and the anonymous referees for their helpful suggestions and corrections on the earlier draft of our paper according to which we improved the content.

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